PART I

INTRODUCTION: MARKETS AND PRICES

CHAPTER 1 PRELIMINARIES

TEACHING NOTES

Chapter 1 covers basic concepts students first saw in their introductory course but could bear some repeating. Since most students will not have read this chapter before the first class, it is a good time to get them talking about some of the concepts presented. You might start by asking for a definition of economics. Make sure to emphasize scarcity and trade-offs. Remind students that the objective of economics is to explain observed phenomena and predict behavior of consumers and firms as economic conditions change. Ask about the differences (and similarities) between microeconomics and macroeconomics and the difference between positive and normative analysis. Review the concept of a market and the role prices play in allocating resources. Discussions of economic theories and models may be a bit abstract at this point in the course, but you can lay the groundwork for a deeper discussion that might take place when you cover consumer behavior in Chapter 3.

Section 1.3 considers real and nominal prices. Given the reliance on dollar prices in the economy, students must understand the difference between real and nominal prices and how to compute real prices. Most students know about the Consumer Price Index, so you might also mention other price indexes such as the Producer Price Index and the Personal Consumption Expenditures (PCE) Price Index, which is the Fed's preferred inflation measure.[1] It is very useful to go over some numerical examples using goods that are in the news and/or that students often purchase such as gasoline, food, textbooks, and a college education.[2]

In general, the first class is a good time to pique student interest in the course. It is also a good time to tell students that they need to work hard to learn how to do economic analysis, and that memorization alone will not get them through the course. Students must learn to think like economists, so encourage them to work lots of problems. Also encourage them to draw graphs neatly and large enough to make them easy to interpret. It always amazes me to see the tiny, poorly drawn graphs some students produce. It is no wonder their answers are often incorrect. You might even suggest they bring a small ruler and colored pencils to class so they can draw good diagrams.

QUESTIONS FOR REVIEW

1. It is often said that a good theory is one that can be refuted by an empirical, data-oriented study. Explain why a theory that cannot be evaluated empirically is not a good theory.

A theory is useful only if it succeeds in explaining and predicting the phenomena it was intended to explain. If a theory cannot be evaluated or tested by comparing its predictions to known facts and data, then we have no idea whether the theory is valid. If we cannot validate the theory, we cannot have any confidence in its predictions, and it is of little use.

[1] The CPI and PPI are reported by the Bureau of Labor Statistics (www.bls.gov). The PCE Price Index is compiled by the Bureau of Economic Analysis in the Commerce Department (www.bea.gov).

[2] The College Board collects data on college tuition (www.collegeboard.com).

2. Which of the following two statements involves positive economic analysis and which normative? How do the two kinds of analysis differ?

a. Gasoline rationing (allocating to each individual a maximum amount of gasoline that can be purchased each year) is a poor social policy because it interferes with the workings of the competitive market system.

Positive economic analysis is concerned with explaining *what is* and predicting *what will be*. Normative economic analysis describes *what ought to be*. Statement (a) is primarily normative because it makes the normative assertion (i.e., a value judgment) that gasoline rationing is "poor social policy." There is also a positive element to statement (a), because it claims that gasoline rationing "interferes with the workings of the competitive market system." This is a prediction that a constraint placed on demand will change the market equilibrium.

b. Gasoline rationing is a policy under which more people are made worse off than are made better off.

Statement (b) is positive because it predicts how gasoline rationing effects people without making a value judgment about the desirability of the rationing policy.

3. Suppose the price of regular-octane gasoline were 20 cents per gallon higher in New Jersey than in Oklahoma. Do you think there would be an opportunity for arbitrage (i.e., that firms could buy gas in Oklahoma and then sell it at a profit in New Jersey)? Why or why not?

Oklahoma and New Jersey represent separate geographic markets for gasoline because of high transportation costs. There would be an opportunity for arbitrage if transportation costs were less than 20 cents per gallon. Then arbitrageurs could make a profit by purchasing gasoline in Oklahoma, paying to transport it to New Jersey and then selling it in New Jersey. If the transportation costs were 20 cents or higher, however, no arbitrage would take place.

4. In Example 1.3, what economic forces explain why the real price of eggs has fallen while the real price of a college education has increased? How have these changes affected consumer choices?

The price and quantity of goods (e.g., eggs) and services (e.g., a college education) are determined by the interaction of supply and demand. The real price of eggs fell from 1970 to 2007 because of either a reduction in demand (e.g., consumers switched to lower-cholesterol food), a reduction in production costs (e.g., improvements in egg production technology), or both. In response, the price of eggs relative to other foods decreased. The real price of a college education rose because of either an increase in demand (e.g., the perceived value of a college education increased, population increased, etc.), an increase in the cost of education (e.g., increase in faculty and staff salaries), or both.

5. Suppose that the Japanese yen rises against the U.S. dollar – that is, it will take more dollars to buy any given amount of Japanese yen. Explain why this increase simultaneously increases the real price of Japanese cars for U.S. consumers and lowers the real price of U.S. automobiles for Japanese consumers.

As the value of the yen grows relative to the dollar, it takes more dollars to purchase a yen, and it takes fewer yen to purchase a dollar. Assume that the costs of production for both Japanese and U.S. automobiles remain unchanged. Then using the new exchange rate, the purchase of a Japanese automobile priced in yen requires more dollars, so the real price of Japanese cars in dollars increases. Similarly, the purchase of a U.S. automobile priced in dollars requires fewer yen, and thus the real price of a U.S automobile in yen decreases.

6. The price of long-distance telephone service fell from 40 cents per minute in 1996 to 22 cents per minute in 1999, a 45-percent (18 cents/40 cents) decrease. The Consumer Price Index increased by 10 percent over this period. What happened to the real price of telephone service?

Let the CPI for 1996 equal 100 and the CPI for 1999 equal 110, which reflects a 10% increase in the overall price level. Now let's find the real price of telephone service (in 1996 dollars) in each year. The real price in 1996 is 40 cents. To find the real price in 1999, divide CPI_{1996} by CPI_{1999} and multiply the result by the nominal price in 1999. The result is (100/110)(22) = 20 cents. The real price therefore fell from 40 to 20 cents, a 50% decline.

EXERCISES

1. Decide whether each of the following statements is true or false and explain why:

a. Fast-food chains like McDonald's, Burger King, and Wendy's operate all over the United States. Therefore the market for fast food is a national market.

This statement is false. People generally buy fast food locally and do not travel large distances across the United States just to buy a cheaper fast food meal. Because there is little potential for arbitrage between fast food restaurants that are located some distance from each other, there are likely to be multiple fast food markets across the country.

b. People generally buy clothing in the city in which they live. Therefore there is a clothing market in, say, Atlanta that is distinct from the clothing market in Los Angeles.

This statement is false. Although consumers are unlikely to travel across the country to buy clothing, they can purchase many items online. In this way, clothing retailers in different cities compete with each other and with online stores such as L.L. Bean. Also, suppliers can easily move clothing from one part of the country to another. Thus, if clothing is more expensive in Atlanta than Los Angeles, clothing companies can shift supplies to Atlanta, which would reduce the price in Atlanta. Occasionally, there may be a market for a specific clothing item in a faraway market that results in a great opportunity for arbitrage, such as the market for blue jeans in the old Soviet Union.

c. Some consumers strongly prefer Pepsi and some strongly prefer Coke. Therefore there is no single market for colas.

This statement is false. Although some people have strong preferences for a particular brand of cola, the different brands are similar enough that they constitute one market. There are consumers who do not have strong preferences for one type of cola, and there are consumers who may have a preference, but who will also be influenced by price. Given these possibilities, the price of cola drinks will not tend to differ by very much, particularly for Coke and Pepsi.

2. The following table shows the average retail price of butter and the Consumer Price Index from 1980 to 2000, scaled so that the CPI = 100 in 1980.

ˇ	1980	1985	1990	1995	2000
CPI	**100**	**130.58**	**158.56**	**184.95**	**208.98**
Retail price of butter (salted, grade AA, per lb.)	**$1.88**	**$2.12**	**$1.99**	**$1.61**	**$2.52**

a. Calculate the real price of butter in 1980 dollars. Has the real price increased/decreased/stayed the same since 1980?

Real price of butter in year t = $\frac{CPI_{1980}}{CPI_t}$ *(nominal price of butter in year t).

ˇ	1980	1985	1990	1995	2000
Real price of butter (1980 $)	$1.88	$1.62	$1.26	$0.87	$1.21

The real price of butter decreased from $1.88 in 1980 to $1.21 in 2000, although it did increase between 1995 and 2000.

b. What is the percentage change in the real price (1980 dollars) from 1980 to 2000?

Real price decreased by $0.67 (1.88 – 1.21 = 0.67). The percentage change in real price from 1980 to 2000 was therefore (–0.67/1.88)*100% = –35.6%.

c. Convert the CPI into 1990 = 100 and determine the real price of butter in 1990 dollars.

To convert the CPI into 1990 = 100, divide the CPI for each year by the CPI for 1990 and multiply that result by 100. Use the formula from part (a) and the new CPI numbers below to find the real price of milk in 1990 dollars.

ˇ	1980	1985	1990	1995	2000
New CPI	63.07	82.35	100	116.64	131.80
Real price of butter (1990 $)	$2.98	$2.57	$1.99	$1.38	$1.91

d. What is the percentage change in the real price (1990 dollars) from 1980 to 2000? Compare this with your answer in (b). What do you notice? Explain.

Real price decreased by $1.07 (2.98 – 1.91 = 1.07). The percentage change in real price from 1980 to 2000 was therefore (–1.07/2.98)*100% = –35.9%. This answer is the same (except for rounding error) as in part (b). It does not matter which year is chosen as the base year when calculating percentage changes in real prices.

3. At the time this book went to print, the minimum wage was $5.85. To find the current value of the CPI, go to http://www.bls.gov/cpi/home.htm. Click on Consumer Price Index-All Urban Consumers (Current Series) and select U.S. All items. This will give you the CPI from 1913 to the present.

a. With these values, calculate the current real minimum wage in 1990 dollars.

The last year of data available when these answers were prepared was 2007. Thus, all calculations are as of 2007. You should update these values for the current year.

Real minimum wage in 2007 = $\frac{CPI_{1990}}{CPI_{2007}}$ * (minimum wage in 2007) = $\frac{130.7}{207.342}$ * $5.85 = $3.69. So, as of 2007, the real minimum wage in 1990 dollars was $3.69.

b. Stated in real 1990 dollars, what is the percentage change in the real minimum wage from 1985 to the present?

The minimum wage in 1985 was $3.35. You can get a complete listing of historical minimum wage rates from the Department of Labor, Employment Standards Administration at http://www.dol.gov/esa/minwage/chart.htm.

Real minimum wage in 1985 = $\frac{CPI_{1990}}{CPI_{1985}}$ * $3.35 = $\frac{130.7}{107.6}$ * $3.35 = $4.07.

The real minimum wage therefore decreased from $4.07 in 1985 to $3.69 in 2007 (all in 1990 dollars). This is a decrease of $4.07 – 3.69 = $0.38, so the percentage change is (–0.38/4.07)*100% = –9.3%.

CHAPTER 2
THE BASICS OF SUPPLY AND DEMAND

TEACHING NOTES

This chapter reviews the basics of supply and demand that students should be familiar with from their introductory economics courses. You may choose to spend more or less time on this chapter depending on how much review your students require. Chapter 2 departs from the standard treatment of supply and demand basics found in most other intermediate microeconomics textbooks by discussing many real-world markets (copper, office space in New York City, wheat, gasoline, natural gas, coffee and others) and teaching students how to analyze these markets with the tools of supply and demand. The real-world applications are intended to show students the relevance of supply and demand analysis, and you may find it helpful to refer to these examples during class.

One of the most common problems students have in supply/demand analysis is confusion between a *movement along a supply or demand curve* and a *shift in the curve*. You should stress the *ceteris paribus* assumption, and explain that all variables except price are held constant along a supply or demand curve. So movements along the demand curve occur *only with changes in price*. When one of the omitted factors changes, the entire supply or demand curve shifts. You might find it useful to make up a simple linear demand function with quantity demanded on the left and the good's price, a competing good's price and income on the right. This gives you a chance to discuss substitutes and complements and also normal and inferior goods. Plug in values for the competing good's price and income and plot the demand curve. Then change, say, the other good's price and plot the demand curve again to show that it shifts. This demonstration helps students understand that the other variables are actually in the demand function and are merely lumped into the intercept term when we draw a demand curve. The same, of course, applies to supply curves as well.

It is important to make the distinction between quantity demanded as a function of price, $Q_D = D(P)$, and the inverse demand function, $P = D^{-1}(Q_D)$, where price is a function of the quantity demanded. Since we plot price on the vertical axis, the inverse demand function is very useful. You can demonstrate this if you use an example as suggested above and plot the resulting demand curves. And, of course, there are "regular" and inverse supply curves as well.

Students also can have difficulties understanding how a market adjusts to a new equilibrium. They often think that the supply and/or demand curves shift as part of the equilibrium process. For example, suppose demand increases. Students typically recognize that price must increase, but some go on to say that supply will also have to increase to satisfy the increased level of demand. This may be a case of confusing an increase in quantity supplied with an increase in supply, but I have seen many students draw a shift in supply, so I try to get this cleared up as soon as possible.

The concept of elasticity, introduced in Section 2.4, is another source of problems. It is important to stress the fact that any elasticity is the ratio of two percentages. So, for example, if a firm's product has a price elasticity of demand of –2, the firm can determine that a 5% increase in price will result in a 10% drop in sales. Use lots of concrete examples to convince students that firms and governments can make important use of elasticity information. A common source of confusion is the negative value for the price elasticity of demand. We often talk about it as if it were a positive number. The book is careful in referring to the "magnitude" of the price elasticity, by which it means the absolute value of the price elasticity, but students may not pick this up on their own. I warn students that I will speak of price elasticities as if they were positive numbers and will say that a good whose elasticity is –2 is more elastic (or greater) than one whose elasticity is –1, even though the mathematically inclined may cringe.

Section 2.6 brings a lot of this material together because elasticities are used to derive demand and supply curves, market equilibria are computed, curves are shifted, and new equilibria are determined. This shows students how we can estimate the quantitative (not just the qualitative) effects of, say, a disruption in oil supply as in Example 2.9.

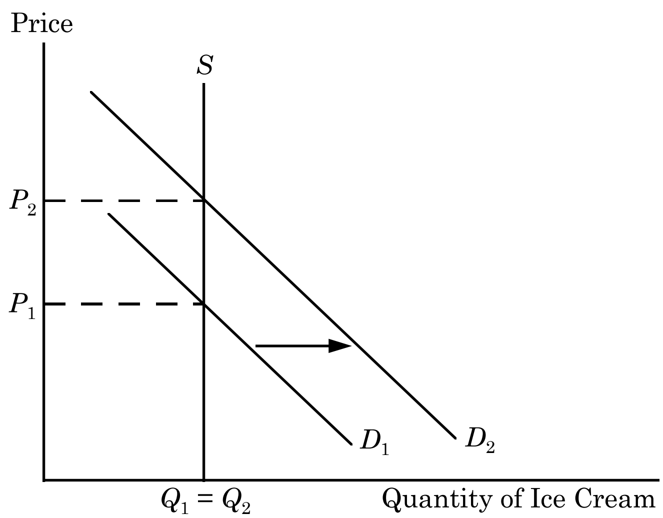
Unfortunately, this section takes some time to cover, especially if your students' algebra is rusty. You'll have to decide whether the benefits outweigh the costs.

Price controls are introduced in Section 2.7. Students usually don't realize the full effects of price controls. They think only of the initial effect on prices without realizing that shortages or surpluses are created, so this is an important topic. However, the coverage here is quite brief. Chapter 9 examines the effects of price controls and other forms of government intervention in much greater detail, so you may want to defer this topic until then.

QUESTIONS FOR REVIEW

1. Suppose that unusually hot weather causes the demand curve for ice cream to shift to the right. Why will the price of ice cream rise to a new market-clearing level?

Suppose the supply of ice cream is completely inelastic in the short run, so the supply curve is vertical as shown below. The initial equilibrium is at price P_1. The unusually hot weather causes the demand curve for ice cream to shift from D_1 to D_2, creating short-run excess demand (i.e., a temporary shortage) at the current price. Consumers will bid against each other for the ice cream, putting upward pressure on the price, and ice cream sellers will react by raising price. The price of ice cream will rise until the quantity demanded and the quantity supplied are equal, which occurs at price P_2.



2. Use supply and demand curves to illustrate how each of the following events would affect the price of butter and the quantity of butter bought and sold:

a. An increase in the price of margarine.

Butter and margarine are substitute goods for most people. Therefore, an increase in the price of margarine will cause people to increase their consumption of butter, thereby shifting the demand curve for butter out from D_1 to D_2 in Figure 2.2.a. This shift in demand causes the equilibrium price of butter to rise from P_1 to P_2 and the equilibrium quantity to increase from Q_1 to Q_2.

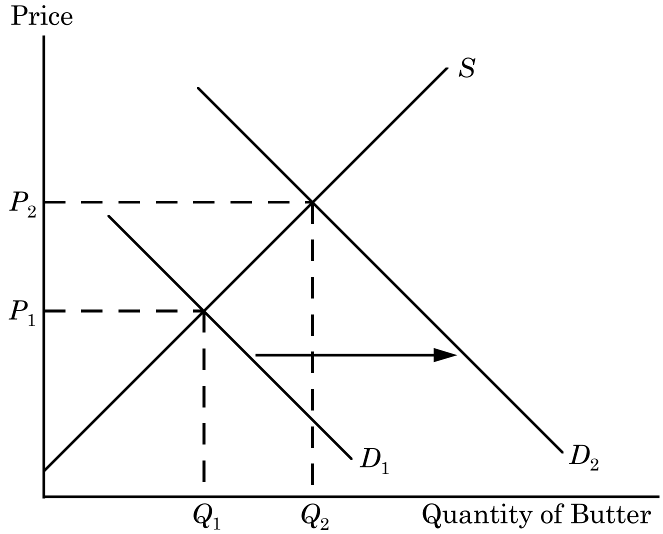


Figure 2.2.a

b. An increase in the price of milk.

Milk is the main ingredient in butter. An increase in the price of milk increases the cost of producing butter, which reduces the supply of butter. The supply curve for butter shifts from S_1 to S_2 in Figure 2.2.b, resulting in a higher equilibrium price, P_2 and a lower equilibrium quantity, Q_2, for butter.

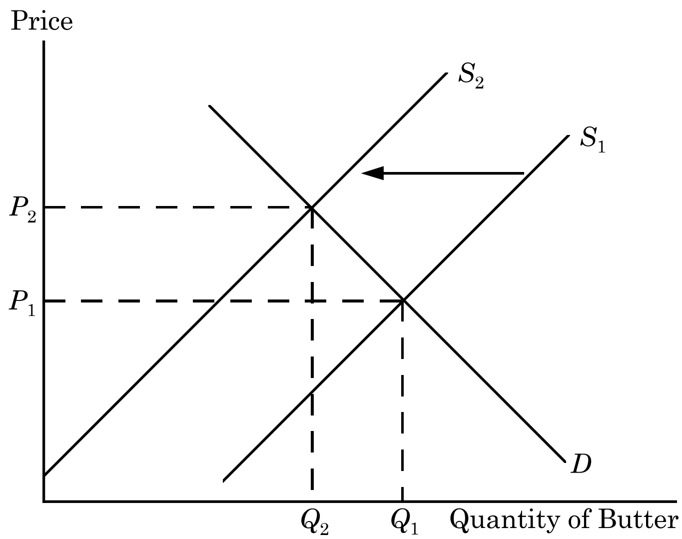


Figure 2.2.b

Note: Butter is in fact made from the fat that is skimmed from milk; thus butter and milk are joint products, and this complicates things. If you take account of this relationship, your answer might change, but it depends on why the price of milk increased. If the increase were caused by an increase in the demand for milk, the equilibrium quantity of milk supplied would increase. With more milk being produced, there would be more milk fat available to make butter, and the price of milk fat would fall.

This would shift the supply curve for butter to the right, resulting in a drop in the price of butter and an increase in the quantity of butter supplied.

c. A decrease in average income levels.

Assuming that butter is a normal good, a decrease in average income will cause the demand curve for butter to decrease (i.e., shift from D_1 to D_2). This will result in a decline in the equilibrium price from P_1 to P_2, and a decline in the equilibrium quantity from Q_1 to Q_2. See Figure 2.2.c.

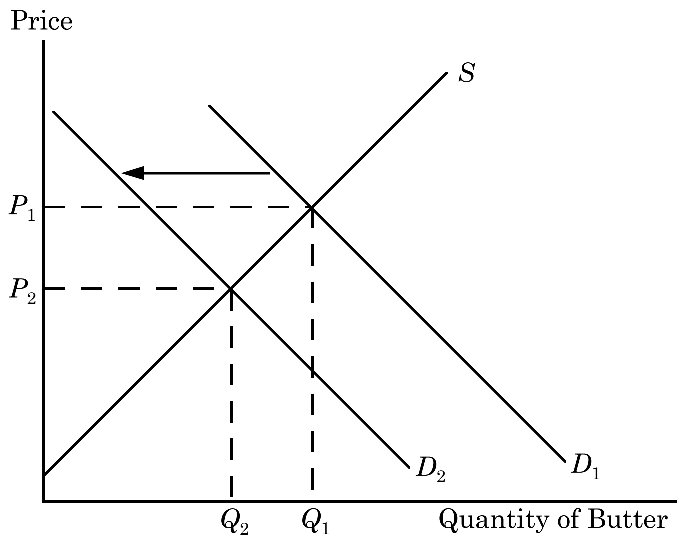


Figure 2.2.c

3. If a 3-percent increase in the price of corn flakes causes a 6-percent decline in the quantity demanded, what is the elasticity of demand?

The elasticity of demand is the percentage change in the quantity demanded divided by the percentage change in the price. The elasticity of demand for corn flakes is therefore

$$E_P^D = \frac{\%\Delta Q}{\%\Delta P} = \frac{-6}{+3} = -2 .$$

4. Explain the difference between a shift in the supply curve and a movement along the supply curve.

A movement along the supply curve occurs when the price of the good changes. A shift of the supply curve is caused by a change in something other than the good's price that results in a change in the quantity supplied at the current price. Some examples are a change in the price of an input, a change in technology that reduces the cost of production and an increase in the number of firms supplying the product.

5. Explain why for many goods, the long-run price elasticity of supply is larger than the short-run elasticity.

The price elasticity of supply is the percentage change in the quantity supplied divided by the percentage change in price. In the short run, an increase in price induces firms to produce more by using their facilities more hours per week, paying workers to work overtime and hiring new workers. Nevertheless, there is a limit to how much firms can produce because they face capacity constraints in the short run. In the long run, however, firms can expand capacity by building new plants and hiring new permanent workers. Also, new firms can enter the market and add their output to total supply. Therefore, the price elasticity of supply is larger in the long run than in the short run.

6. Why do long-run elasticities of demand differ from short-run elasticities? Consider two goods: paper towels and televisions. Which is a durable good? Would you expect the price elasticity of demand for paper towels to be larger in the short run or in the long run? Why? What about the price elasticity of demand for televisions?

Long-run and short-run elasticities differ based on how rapidly consumers respond to price changes and how many substitutes are available. If the price of paper towels, a non-durable good, were to increase, consumers might react only minimally in the short run- because it takes time for people to change their consumption habits. In the long run, however, consumers might learn to use other products such as sponges or kitchen towels instead of paper towels. In this case, then, the price elasticity would be larger in the long run than in the short run. In contrast, the quantity demanded of durable goods, such as televisions, might change dramatically in the short run following a price change. For example, the initial result of a price increase for televisions would cause consumers to delay purchases because they could keep using their current TVs longer. Eventually consumers would replace their televisions as they wore out or became obsolete. Therefore, we expect the demand for durables to be more elastic in the short run than in the long run.

7. Are the following statements true or false? Explain your answers.

a. The elasticity of demand is the same as the slope of the demand curve.

False. Elasticity of demand is the percentage change in quantity demanded divided by the percentage change in the price of the product. In contrast, the slope of the demand curve is the change in quantity demanded (in units) divided by the change in price (typically in dollars). The difference is that elasticity uses percentage changes while the slope is based on changes in the number of units and number of dollars.

b. The cross-price elasticity will always be positive.

False. The cross price elasticity measures the percentage change in the quantity demanded of one good due to a one percent change in the price of another good. This elasticity will be positive for substitutes (an increase in the price of hot dogs is likely to cause an increase in the quantity demanded of hamburgers) and negative for complements (an increase in the price of hot dogs is likely to cause a decrease in the quantity demanded of hot dog buns).

c. The supply of apartments is more inelastic in the short run than the long run.

True. In the short run it is difficult to change the supply of apartments in response to a change in price. Increasing the supply requires constructing new apartment buildings, which can take a year or more. Therefore, the elasticity of supply is more inelastic in the short run than in the long run, or said another way, the elasticity of supply is less elastic in the short run than in the long run.

8. Suppose the government regulates the prices of beef and chicken and sets them below their market-clearing levels. Explain why shortages of these goods will develop and what factors will determine the sizes of the shortages. What will happen to the price of pork? Explain briefly.

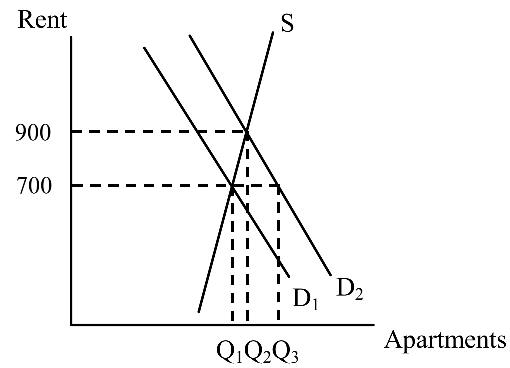
If the price of a commodity is set below its market-clearing level, the quantity that firms are willing to supply is less than the quantity that consumers wish to purchase. The extent of the resulting shortage depends on the elasticities of demand and supply as well as the amount by which the regulated price is set below the market-clearing price. For instance, if both supply and demand are elastic, the shortage is larger than if both are inelastic, and if the regulated price is substantially below the market-clearing price, the shortage is larger than if the regulated price is only slightly below the market-clearing price.

Factors such as the willingness of consumers to eat less meat and the ability of farmers to reduce the size of their herds/flocks will determine the relevant elasticities. Customers whose demands for beef and chicken are not met because of the shortages will want to purchase substitutes like pork. This increases the demand for pork (i.e., shifts demand to the right), which results in a higher price for pork.

9. The city council of a small college town decides to regulate rents in order to reduce student living expenses. Suppose the average annual market-clearing rent for a two-bedroom apartment had been $700 per month, and rents were expected to increase to $900 within a year. The city council limits rents to their current $700-per-month level.

a. Draw a supply and demand graph to illustrate what will happen to the rental price of an apartment after the imposition of rent controls.

Initially demand is D_1 and supply is S, so the equilibrium rent is $700 and Q_1 apartments are rented. Without regulation, demand was expected to increase to D_2, which would have raised rent to $900 and resulted in Q_2 apartment rentals. Under the city council regulation, however, the rental price stays at the old equilibrium level of $700 per month. After demand increases to D_2, only Q_1 apartments will be supplied while Q_3 will be demanded. There will be a shortage of $Q_3 - Q_1$ apartments.

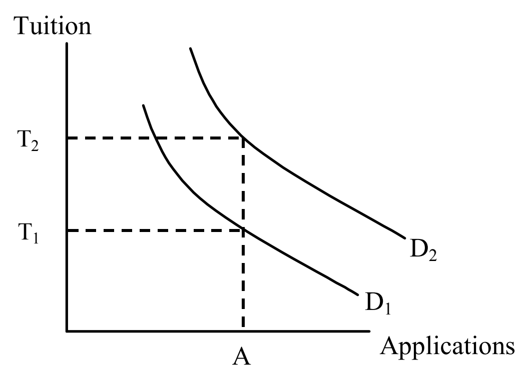


b. Do you think this policy will benefit all students? Why or why not?

No. It will benefit those students who get an apartment, although these students may find that the cost of searching for an apartment is higher given the shortage of apartments. Those students who do not get an apartment may face higher costs as a result of having to live outside the college town. Their rent may be higher and their transportation costs will be higher, so they will be worse off as a result of the policy.

10. In a discussion of tuition rates, a university official argues that the demand for admission is completely price inelastic. As evidence, she notes that while the university has doubled its tuition (in real terms) over the past 15 years, neither the number nor quality of students applying has decreased. Would you accept this argument? Explain briefly. (*Hint*: The official makes an assertion about the demand for admission, but does she actually observe a demand curve? What else could be going on?)

I would not accept this argument. The university official assumes that demand has remained stable (i.e., the demand curve has not shifted) over the 15-year period. This seems very unlikely. Demand for college educations has increased over the years for many reasons – real incomes have increased, population has increased, the perceived value of a college degree has increased, etc. What has probably happened is that tuition doubled from T_1 to T_2, but demand also increased from D_1 to D_2 over the 15 years, and the two effects have offset each other. The result is that the quantity (and quality) of applications has remained steady at A. The demand curve is not perfectly inelastic as the official asserts.



11. Suppose the demand curve for a product is given by Q = 10 – 2P + P_S, where P is the price of the product and P_S is the price of a substitute good. The price of the substitute good is $2.00.

a. Suppose P = $1.00. What is the price elasticity of demand? What is the cross-price elasticity of demand?

Find quantity demanded when P = $1.00 and P_S = $2.00. Q = 10 – 2(1) + 2 = 10. Price elasticity of demand = $\frac{P}{Q}\frac{\Delta Q}{\Delta P}=\frac{1}{10}(-2)=-\frac{2}{10}=-0.2$.

Cross-price elasticity of demand = $\frac{P_S}{Q}\frac{\Delta Q}{\Delta P_S}=\frac{2}{10}(1)=0.2$.

b. Suppose the price of the good, P, goes to $2.00. Now what is the price elasticity of demand? What is the cross-price elasticity of demand?

When P = $2.00. Q = 10 – 2(2) + 2 = 8.

Price elasticity of demand = $\frac{P}{Q}\frac{\Delta Q}{\Delta P}=\frac{2}{8}(-2)=-\frac{4}{8}=-0.5$.

Cross-price elasticity of demand = $\frac{P_S}{Q}\frac{\Delta Q}{\Delta P_S}=\frac{2}{8}(1)=0.25$.

12. Suppose that rather than the declining demand assumed in Example 2.8, a decrease in the cost of copper production causes the supply curve to shift to the right by 40 percent. How will the price of copper change?

If the supply curve shifts to the right by 40% then the new quantity supplied will be 140 percent of the old quantity supplied at every price. The new supply curve is therefore the old supply curve multiplied by 1.4.

Q_S' = 1.4 (–6 + 9P) = –8.4 + 12.6P. To find the new equilibrium price of copper, set the new supply equal to demand. Thus, –8.4 + 12.6P = 18 – 3P. Solving for price results in P = $1.69 per pound for the new equilibrium price. The price decreased by 31 cents per pound, from $2.00 to $1.69.

13. Suppose the demand for natural gas is perfectly inelastic. What would be the effect, if any, of natural gas price controls?

If the demand for natural gas is perfectly inelastic, the demand curve is vertical. Consumers will demand the same quantity regardless of price. In this case, price controls will have no effect on the quantity demanded, but they will still cause a shortage if the supply curve is upward sloping and the regulated price is set below the market-clearing price, because suppliers will produce less natural gas than consumers wish to purchase.

EXERCISES

1. Suppose the demand curve for a product is given by Q = 300 – 2P + 4I, where I is average income measured in thousands of dollars. The supply curve is Q = 3P – 50.

a. If I = 25, find the market clearing price and quantity for the product.

Given I = 25, the demand curve becomes Q = 300 – 2P + 4(25), or Q = 400 – 2P. Setting demand equal to supply we can solve for P and then Q:

$$400 - 2P = 3P - 50$$

$$P = 90$$

$$Q = 220.$$

b. If I = 50, find the market clearing price and quantity for the product.

Given I = 50, the demand curve becomes Q = 300 – 2P + 4(50), or Q = 500 – 2P. Setting demand equal to supply we can solve for P and then Q:

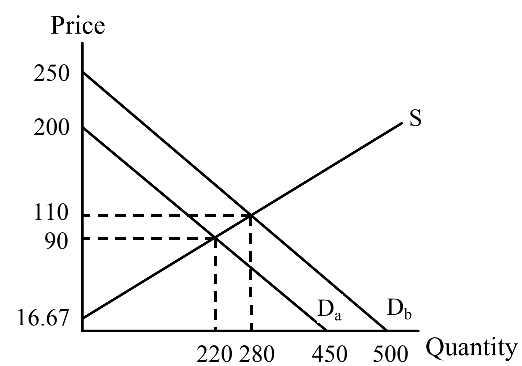
$$500 - 2P = 3P - 50$$

$$P = 110$$

$$Q = 280.$$

c. Draw a graph to illustrate your answers.

It is easier to draw the demand and supply curves if you first solve for the inverse demand and supply functions, i.e., solve the functions for P. Demand in part (a) is P = 200 – 0.5Q and supply is P = 16.67 + 0.333Q. These are shown on the graph as D_a and S. Equilibrium price and quantity are found at the intersection of these demand and supply curves. When the income level increases in part (b), the demand curve shifts up and to the right. Inverse demand is P = 250 – 0.5Q and is labeled D_b. The intersection of the new demand curve and original supply curve is the new equilibrium point.



2. Consider a competitive market for which the quantities demanded and supplied (per year) at various prices are given as follows:

Price (Dollars)	Demand (Millions)	Supply (Millions)
60	22	14
80	20	16
100	18	18
120	16	20

a. Calculate the price elasticity of demand when the price is \$80 and when the price is \$100.

$$E_D = \frac{\frac{\Delta Q_D}{Q_D}}{\frac{\Delta P}{P}} = \frac{P}{Q_D}\frac{\Delta Q_D}{\Delta P}.$$

With each price increase of \$20, the quantity demanded decreases by 2 million. Therefore,

$$\left(\frac{\Delta Q_D}{\Delta P}\right) = \frac{-2}{20} = -0.1.$$

At $P = 80$, quantity demanded is 20 million and thus

$$E_D = \left(\frac{80}{20}\right)(-0.1) = -0.40.$$

Similarly, at $P = 100$, quantity demanded equals 18 million and

$$E_D = \left(\frac{100}{18}\right)(-0.1) = -0.56.$$

b. Calculate the price elasticity of supply when the price is \$80 and when the price is \$100.

$$E_S = \frac{\frac{\Delta Q_S}{Q_S}}{\frac{\Delta P}{P}} = \frac{P}{Q_S}\frac{\Delta Q_S}{\Delta P}.$$

With each price increase of \$20, quantity supplied increases by 2 million. Thus,

$$\left(\frac{\Delta Q_S}{\Delta P}\right) = \frac{2}{20} = 0.1.$$

At $P = 80$, quantity supplied is 16 million and

$$E_S = \left(\frac{80}{16}\right)(0.1) = 0.5.$$

Similarly, at $P = 100$, quantity supplied equals 18 million and

$$E_S = \left(\frac{100}{18}\right)(0.1) = 0.56.$$

c. What are the equilibrium price and quantity?

The equilibrium price is the price at which the quantity supplied equals the quantity demanded. As we see from the table, the equilibrium price is P* = \$100 and the equilibrium quantity is Q* = 18 million.

d. Suppose the government sets a price ceiling of \$80. Will there be a shortage, and if so, how large will it be?

With a price ceiling of \$80, price cannot be above \$80, so the market cannot reach its equilibrium price of \$100. At \$80, consumers would like to buy 20 million, but producers will supply only 16 million. This will result in a shortage of 4 million.

3. Refer to Example 2.5 (page 38) on the market for wheat. In 1998, the total demand for U.S. wheat was Q = 3244 – 283P and the domestic supply was Q_S = 1944 + 207P. At the end of 1998, both Brazil and Indonesia opened their wheat markets to U.S. farmers. Suppose that these new markets add 200 million bushels to U.S. wheat demand. What will be the free-market price of wheat and what quantity will be produced and sold by U.S. farmers?

► **Note**: The answer at the end of the book (first printing) used the wrong demand curve to find the new equilibrium quantity. The correct answer is given below.

If Brazil and Indonesia add 200 million bushels of wheat to U.S. wheat demand, the new demand curve will be $Q + 200$, or

$$Q_D = (3244 - 283P) + 200 = 3444 - 283P.$$

Equate supply and the new demand to find the new equilibrium price.

$$1944 + 207P = 3444 - 283P, \text{ or}$$

$$490P = 1500, \text{ and thus } P = \$3.06 \text{ per bushel.}$$

To find the equilibrium quantity, substitute the price into either the supply or demand equation. Using demand,

$$Q_D = 3444 - 283(3.06) = 2578 \text{ million bushels.}$$

4. A vegetable fiber is traded in a competitive world market, and the world price is \$9 per pound. Unlimited quantities are available for import into the United States at this price. The U.S. domestic supply and demand for various price levels are shown as follows:

Price	**U.S. Supply (Million Lbs.)**	**U.S. Demand (Million Lbs.)**
3	**2**	**34**
6	**4**	**28**
9	**6**	**22**
12	**8**	**16**
15	**10**	**10**
18	**12**	**4**

a. What is the equation for demand? What is the equation for supply?

The equation for demand is of the form Q = a – bP. First find the slope, which is $\frac{\Delta Q}{\Delta P} = \frac{-6}{3} = -2 = -b$. You can figure this out by noticing that every time price increases by 3, quantity demanded falls by 6 million pounds. Demand is now Q = a – 2P. To find a, plug in any of the price and quantity demanded points from the table. For example: Q = 34 = a – 2(3) so that a = 40 and demand is Q = 40 – 2P.

The equation for supply is of the form Q = c + dP. First find the slope, which is $\frac{\Delta Q}{\Delta P} = \frac{2}{3} = d$. You can figure this out by noticing that every time price increases by 3, quantity supplied increases by 2 million pounds. Supply is now $Q = c + \frac{2}{3}P$. To find c, plug in any of the price and quantity supplied points from the table. For example: $Q = 2 = c + \frac{2}{3}(3)$ so that c = 0 and supply is $Q = \frac{2}{3}P$.

b. At a price of \$9, what is the price elasticity of demand? What is it at a price of \$12?

Elasticity of demand at P = 9 is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{9}{22}(-2) = \frac{-18}{22} = -0.82$.

Elasticity of demand at P = 12 is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{12}{16}(-2) = \frac{-24}{16} = -1.5$.

c. What is the price elasticity of supply at \$9? At \$12?

Elasticity of supply at P = 9 is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{9}{6}\left(\frac{2}{3}\right) = \frac{18}{18} = 1.0$.

Elasticity of supply at P = 12 is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{12}{8}\left(\frac{2}{3}\right) = \frac{24}{24} = 1.0$.

d. In a free market, what will be the U.S. price and level of fiber imports?

With no restrictions on trade, the price in the United States will be the same as the world price, so P = \$9. At this price, the domestic supply is 6 million lbs., while the domestic demand is 22 million lbs. Imports make up the difference and are 16 million lbs.

5. Much of the demand for U.S. agricultural output has come from other countries. In 1998, the total demand for wheat was $Q = 3244 - 283P$. Of this, total domestic demand was $Q_D = 1700 - 107P$, and domestic supply was $Q_S = 1944 + 207P$. Suppose the export demand for wheat falls by 40 percent.

a. U.S. farmers are concerned about this drop in export demand. What happens to the free-market price of wheat in the United States? Do the farmers have much reason to worry?

Before the drop in export demand, the market equilibrium price is found by setting total demand equal to domestic supply:

$$3244 - 283P = 1944 + 207P, \text{ or}$$

$$P = \$2.65.$$

Export demand is the difference between total demand and domestic demand: $Q = 3244 - 283P$ minus $Q_D = 1700 - 107P$. So export demand is originally $Q_e = 1544 - 176P$. After the 40 percent drop, export demand is only 60 percent of the original export demand. The new export demand is therefore, $Q'_e = 0.6Q_e = 0.6(1544 - 176P) = 926.4 - 105.6P$. Graphically, export demand has pivoted inward as illustrated in the figure below.

The new total demand becomes

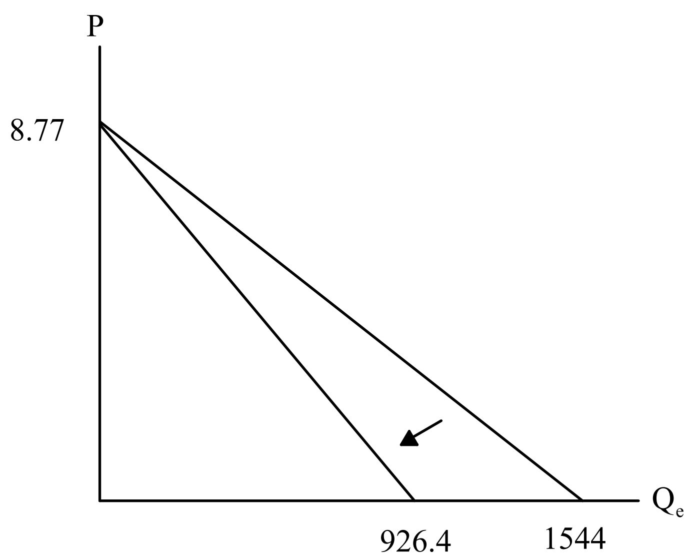
$$Q' = Q_D + Q'_e = (1700 - 107P) + (926.4 - 105.6P) = 2626.4 - 212.6P.$$

Equating total supply and the new total demand,

$$1944 + 207P = 2626.4 - 212.6P, \text{ or}$$

$$P = \$1.63,$$

which is a significant drop from the original market-clearing price of $2.65 per bushel. At this price, the market-clearing quantity is about Q = 2281 million bushels. Total revenue has decreased from about $6609 million to $3718 million, so farmers have a lot to worry about.



b. Now suppose the U.S. government wants to buy enough wheat to raise the price to $3.50 per bushel. With the drop in export demand, how much wheat would the government have to buy? How much would this cost the government?

With a price of $3.50, the market is not in equilibrium. Quantity demanded and supplied are

$$Q' = 2626.4 - 212.6(3.50) = 1882.3, \text{ and}$$

$$Q_S = 1944 + 207(3.50) = 2668.5.$$

Excess supply is therefore 2668.5 – 1882.3 = 786.2 million bushels. The government must purchase this amount to support a price of $3.50, and will have to spend $3.50(786.2 million) = $2751.7 million.

6. The rent control agency of New York City has found that aggregate demand is $Q_D = 160 - 8P$. Quantity is measured in tens of thousands of apartments. Price, the average monthly rental rate, is measured in hundreds of dollars. The agency also noted that the increase in *Q* at lower *P* results from more three-person families coming into the city from Long Island and demanding apartments. The city's board of realtors acknowledges that this is a good demand estimate and has shown that supply is $Q_S = 70 + 7P$.

a. If both the agency and the board are right about demand and supply, what is the free-market price? What is the change in city population if the agency sets a maximum average monthly rent of $300 and all those who cannot find an apartment leave the city?

Set supply equal to demand to find the free-market price for apartments:

$$160 - 8P = 70 + 7P, \text{ or } P = 6,$$

which means the rental price is $600 since price is measured in hundreds of dollars. Substituting the equilibrium price into either the demand or supply equation to determine the equilibrium quantity:

$$Q_D = 160 - 8(6) = 112$$

and

$$Q_S = 70 + 7(6) = 112.$$

The quantity of apartments rented is 1,120,000 since Q is measured in tens of thousands of apartments. If the rent control agency sets the rental rate at $300, the quantity supplied would be 910,000 ($Q_S = 70 + 7(3) = 91$), a decrease of 210,000 apartments from the free-market equilibrium. Assuming three people per family per apartment, this would imply a loss in city population of 630,000 people. Note: At the $300 rental rate, the demand for apartments is 1,360,000 units, and the resulting shortage is 450,000 units (1,360,000 – 910,000). However, excess demand (the shortage) and lower quantity demanded are not the same concept. The shortage of 450,000 units is the difference between the number of apartments demanded at the new lower price (including the number demanded by new people who would have moved into the city), and the number supplied at the lower price. But these new people will not actually move into the city because the apartments are not available. Therefore, the city population will fall by 630,000, which is due to the drop in the number of apartments available from 1,120,000 (the old equilibrium value) to 910,000.

b. Suppose the agency bows to the wishes of the board and sets a rental of $900 per month on all apartments to allow landlords a "fair" rate of return. If 50 percent of any long-run increases in apartment offerings come from new construction, how many apartments are constructed?

At a rental rate of $900, the demand for apartments would be 160 – 8(9) = 88, or 880,000 units, which is 240,000 fewer apartments than the original free-market equilibrium number of 1,120,000. Therefore, no new apartments would be constructed.

7. In 1998, Americans smoked 470 billion cigarettes, or 23.5 billion packs of cigarettes. The average retail price was $2 per pack. Statistical studies have shown that the price elasticity of demand is –0.4, and the price elasticity of supply is 0.5. Using this information, derive linear demand and supply curves for the cigarette market.

Let the demand curve be of the form Q = a – bP and the supply curve be of the form Q = c + dP, where a, b, c, and d are positive constants. To begin, recall the formula for the price elasticity of demand

$$E_P^D = \frac{P}{Q}\frac{\Delta Q}{\Delta P}.$$

We know the demand elasticity is –0.4, P = 2, and Q = 23.5, which means we can solve for the slope, –b, which is ΔQ/ΔP in the above formula.

$$-0.4 = \frac{2}{23.5}\frac{\Delta Q}{\Delta P}$$

$$\frac{\Delta Q}{\Delta P} = -0.4\left(\frac{23.5}{2}\right) = -4.7 = -b.$$

To find the constant a, substitute for Q, P, and b in the demand function to get 23.5 = a – 4.7(2) and a = 32.9.

The equation for demand is therefore Q = 32.9 – 4.7P.

To find the supply curve, recall the formula for the elasticity of supply and follow the same method as above:

$$E_P^S = \frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

$$0.5 = \frac{2}{23.5}\frac{\Delta Q}{\Delta P}$$

$$\frac{\Delta Q}{\Delta P} = 0.5\left(\frac{23.5}{2}\right) = 5.875 = d.$$

To find the constant c, substitute for Q, P, and d in the supply function to get 23.5 = c + 5.875(2) and c = 11.75. The equation for supply is therefore Q = 11.75 + 5.875P.

8. In Example 2.8 we examined the effect of a 20-percent decline in copper demand on the price of copper, using the linear supply and demand curves developed in Section 2.6. Suppose the long-run price elasticity of copper demand were –0.75 instead of –0.5.

a. Assuming, as before, that the equilibrium price and quantity are *P = $2 per pound and *Q** = 12 million metric tons per year, derive the linear demand curve consistent with the smaller elasticity.**

Following the method outlined in Section 2.6, we solve for a and b in the demand equation $Q_D = a - bP$. Because –b is the slope, we can use –b rather than ΔQ/ΔP in the elasticity formula. Therefore, $E_D = -b\left(\frac{P^*}{Q^*}\right)$. Here E_D = –0.75 (the long-run price elasticity), $P^* = 2$ and $Q^* = 12$. Solving for b,

$$-0.75 = -b\left(\frac{2}{12}\right), \text{ or } b = 0.75(6) = 4.5.$$

To find the intercept, we substitute for b, Q_D (= Q^*), and P (= P^*) in the demand equation:

12 = a – 4.5(2), or a = 21.

The linear demand equation is therefore

$$Q_D = 21 - 4.5P.$$

b. Using this demand curve, recalculate the effect of a 20-percent decline in copper demand on the price of copper.

The new demand is 20 percent below the original (using our convention that quantity demanded is reduced by 20% at every price); therefore, multiply demand by 0.8 because the new demand is 80 percent of the original demand:

$$Q'_D = (0.8)(21 - 4.5P) = 16.8 - 3.6P.$$

Equating this to supply,

$$16.8 - 3.6P = -6 + 9P, \text{ or}$$

$$P = \$1.81.$$

With the 20-percent decline in demand, the price of copper falls from \$2.00 to \$1.81 per pound. The decrease in demand therefore leads to a drop in price of 19 cents per pound, a 9.5 percent decline.

9. In Example 2.8 (page 52), we discussed the recent increase in world demand for copper, due in part to China's rising consumption.

a. Using the original elasticities of demand and supply (i.e. $E_S = 1.5$ and $E_D = -0.5$), calculate the effect of a 20-percent *increase* in copper demand on the price of copper.

The original demand is Q = 18 – 3P and supply is Q = –6 + 9P as shown on page 51. The 20-percent increase in demand means that the new demand is 120 percent of the original demand, so the new demand is $Q'_D = 1.2Q$. $Q'_D = (1.2)(18 - 3P) = 21.6 - 3.6P$. The new equilibrium is where Q'_D equals the original supply:

$$21.6 - 3.6P = -6 + 9P.$$

The new equilibrium price is P* = \$2.19 per pound. An increase in demand of 20 percent, therefore, increases price by 19 cents per pound, or 9.5 percent.

b. Now calculate the effect of this increase in demand on the equilibrium quantity, Q*.

Using the new price of \$2.19 in the supply curve, the new equilibrium quantity is Q* = –6 + 9(2.19) = 13.71 million metric tons (mmt) per year, an increase of 1.71 mmt per year. Except for rounding, you get the same result by plugging the new price of \$2.19 into the new demand curve. So an increase in demand of 20 percent increases quantity by 1.71 mmt per year, or 14.3 percent.

c. As we discussed in Example 2.8, the U.S. production of copper declined between 2000 and 2003. Calculate the effect on the equilibrium price and quantity of *both* a 20-percent increase in copper demand (as you just did in part a) *and* of a 20-percent decline in copper supply.

The new supply of copper falls (shifts to the left) to 80 percent of the original, so $Q'_S = 0.8Q = (0.8)(-6 + 9P) = -4.8 + 7.2P$. The new equilibrium is where $Q'_D = Q'_S$.

$$21.6 - 3.6P = -4.8 + 7.2P$$

The new equilibrium price is P* = \$2.44 per pound. Plugging this price into the new supply equation, the new equilibrium quantity is Q* = –4.8 + 7.2(2.44) = 12.77 million metric tons per year. Except for rounding, you get the same result if you substitute the new price into the new demand equation. The combined effect of a 20-percent increase in demand and a 20-percent decrease in supply is that price increases by 44 cents per pound, or 22 percent, and quantity increases by 0.77 mmt per year, or 6.4 percent, compared to the original equilibrium.

10. Example 2.9 (page 54) analyzes the world oil market. Using the data given in that example:

a. Show that the short-run demand and competitive supply curves are indeed given by

$$D = 35.5 - 0.03P$$

$$S_C = 18 + 0.04P.$$

The competitive (non-OPEC) quantity supplied is $S_c = Q^* = 20$. The general form for the linear competitive supply equation is $S_C = c + dP$. We can write the short-run supply elasticity as $E_S = d(P^*/Q^*)$. Since $E_S = 0.10$, $P^* = \$50$, and $Q^* = 20$, $0.10 = d(50/20)$. Hence $d = 0.04$. Substituting for d, S_c, and P in the supply equation, $c = 18$, and the short-run competitive supply equation is $S_c = 18 + 0.04P$.

Similarly, world demand is $D = a - bP$, and the short-run demand elasticity is $E_D = -b(P^*/Q^*)$, where Q^* is total world demand of 34. Therefore, $-0.05 = -b(50/34)$, and $b = 0.034$, or 0.03 rounded off. Substituting $b = 0.03$, $D = 34$, and $P = 50$ in the demand equation gives $34 = a - 0.03(50)$, so that $a = 35.5$. Hence the short-run world demand equation is $D = 35.5 - 0.03P$.

b. Show that the long-run demand and competitive supply curves are indeed given by

$$D = 47.5 - 0.27P$$

$$S_C = 12 + 0.16P.$$

Do the same calculations as above but now using the long-run elasticities, $E_S = 0.4$ and $E_D = -0.4$: $E_S = d(P^*/Q^*)$ and $E_D = -b(P^*/Q^*)$, implying $0.4 = d(50/20)$ and $-0.4 = -b(50/34)$. So $d = 0.16$ and $b = 0.27$.

Next solve for c and a: $S_c = c + dP$ and $D = a - bP$, implying $20 = c + 0.16(50)$ and $34 = a - 0.27(50)$. So $c = 12$ and $a = 47.5$.

c. In Example 2.9 we examined the impact on price of a disruption of oil from Saudi Arabia. Suppose that instead of a decline in supply, OPEC production *increases* by 2 billion barrels per year (bb/yr) because the Saudis open large new oil fields. Calculate the effect of this increase in production on the supply of oil in both the short run and the long run.

OPEC's supply increases from 14 bb/yr to 16 bb/yr as a result. Add 16 bb/yr to the short-run and long-run competitive supply equations. The new total supply equations are:

Short-run: $S_T{'} = 16 + S_c = 16 + 18 + 0.04P = 34 + 0.04P$, and

Long-run: $S_T{''} = 16 + S_c = 16 + 12 + 0.16P = 28 + 0.16P$.

These are equated with short-run and long-run demand, so that:

$34 + 0.04P = 35.5 - 0.03P$, implying that $P = \$21.43$ in the short run, and

$28 + 0.16P = 47.5 - 0.27P$, implying that $P = \$45.35$ in the long run.

11. Refer to Example 2.10 (page 59), which analyzes the effects of price controls on natural gas.

a. Using the data in the example, show that the following supply and demand curves describe the market for natural gas in 2005 – 2007:

$$\textbf{Supply: } Q = 15.90 + 0.72P_G + 0.05P_O$$

$$\textbf{Demand: } Q = 0.02 - 1.8P_G + 0.69P_O$$

Also, verify that if the price of oil is $50, these curves imply a free-market price of $6.40 for natural gas.

To solve this problem, apply the analysis of Section 2.6 using the definition of cross-price elasticity of demand given in Section 2.4. For example, the cross-price-elasticity of demand for natural gas with respect to the price of oil is:

$$E_{GO} = \left(\frac{\Delta Q_G}{\Delta P_O}\right)\left(\frac{P_O}{Q_G}\right).$$

$\left(\frac{\Delta Q_G}{\Delta P_O}\right)$ is the change in the quantity of natural gas demanded because of a small change in the price of oil, and for linear demand equations, it is constant. If we represent demand as $Q_G = a - bP_G + eP_O$ (notice that income is held constant), then $\left(\frac{\Delta Q_G}{\Delta P_O}\right) = e$. Substituting this into the cross-price elasticity, $E_{GO} = e\left(\frac{P_O^*}{Q_G^*}\right)$, where P_O^* and Q_G^* are the equilibrium price and quantity. We know that P_O^* = \$50 and Q_G^* = 23 trillion cubic feet (Tcf). Solving for e,

$$1.5 = e\left(\frac{50}{23}\right), \text{ or } e = 0.69.$$

Similarly, representing the supply equation as $Q_G = c + dP_G + gP_O$, the cross-price elasticity of supply is $g\left(\frac{P_O^*}{Q_G^*}\right)$, which we know to be 0.1. Solving for g, $0.1 = g\left(\frac{50}{23}\right)$, or g = 0.5.

We know that $E_S = 0.2$, $P_G^* = 6.40$, and $Q^* = 23$. Therefore, $0.2 = d\left(\frac{6.40}{23}\right)$, or $d = 0.72$.

Also, $E_D = -0.5$, so $-0.5 = -b\left(\frac{6.40}{23}\right)$, and thus $b = 1.8$.

By substituting these values for d, g, b, and e into our linear supply and demand equations, we may solve for c and a:

$$23 = c + .72(6.40) + .05(50), \text{ so } c = 15.9, \text{ and}$$
$$23 = a - 1.8(6.40) + 0.69(50), \text{ so that } a = 0.02.$$

Therefore, the supply and demand curves for natural gas are as given. If the price of oil is $50, these curves imply a free-market price of $6.40 for natural gas as shown below. Substitute the price of oil in the supply and demand equations. Then set supply equal to demand and solve for the price of gas.

$$15.9 + 0.72P_G + .05(50) = 0.02 - 1.8P_G + 0.69(50)$$

$$18.4 + 0.72P_G = 34.52 - 1.8P_G$$

$$P_G = \$6.40.$$

b. Suppose the regulated price of gas were $4.50 per thousand cubic feet instead of $3.00. How much excess demand would there have been?

With a regulated price of $4.50 for natural gas and the price of oil equal to $50 per barrel,

$$\text{Demand: } Q_D = 0.02 - 1.8(4.50) + 0.69(50) = 26.4, \text{ and}$$
$$\text{Supply: } Q_S = 15.9 + 0.72(4.50) + 0.05(50) = 21.6.$$

With a demand of 26.4 Tcf and a supply of 21.6 Tcf, there would be an excess demand (i.e., a shortage) of 4.8 Tcf.

c. Suppose that the market for natural gas remained unregulated. If the price of oil had increased from $50 to $100, what would have happened to the free-market price of natural gas?

In this case

$$\text{Demand: } Q_D = 0.02 - 1.8P_G + 0.69(100) = 69.02 - 1.8P_G, \text{ and}$$
$$\text{Supply: } Q_S = 15.9 + 0.72P_G + 0.05(100) = 20.9 + 0.72P_G.$$

Equating supply and demand and solving for the equilibrium price,

$$20.9 + 0.72P_G = 69.02 - 1.8P_G, \text{ or } P_G = \$19.10.$$

The price of natural gas would have almost tripled from $6.40 to $19.10.

12. The table below shows the retail price and sales for instant coffee and roasted coffee for 1997 and 1998.

Year	Retail Price of Instant Coffee ($/Lb)	Sales of Instant Coffee (Million Lbs)	Retail Price of Roasted Coffee ($/Lb)	Sales of Roasted Coffee (Million Lbs)
1997	**10.35**	**75**	**4.11**	**820**
1998	**10.48**	**70**	**3.76**	**850**

a. Using these data alone, estimate the short-run price elasticity of demand for roasted coffee. Derive a linear demand curve for roasted coffee.

To find elasticity, first estimate the slope of the demand curve:

$$\frac{\Delta Q}{\Delta P} = \frac{820 - 850}{4.11 - 3.76} = \frac{-30}{0.35} = -85.7$$

Given the slope, we can now estimate elasticity using the price and quantity data from the above table. Assuming the demand curve is linear, the elasticity will differ in 1997 and 1998, because price and quantity are different. We can calculate the elasticities at both points and also find the arc elasticity at the average point between the two years:

$$E_P^{97} = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{4.11}{820}(-85.7) = -0.43$$

$$E_P^{98} = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{3.76}{850}(-85.7) = -0.38$$

$$E_P^{ARC} = \frac{\frac{P_{97}+P_{98}}{2}}{\frac{Q_{97}+Q_{98}}{2}}\frac{\Delta Q}{\Delta P} = \frac{3.935}{835}(-85.7) = -0.40$$

To derive the demand curve for roasted coffee, Q = a – bP, note that the slope of the demand curve is –85.7 = –b. To find the coefficient a, use either of the data points from the table above so that 820 = a – 85.7(4.11) or 850 = a – 85.7(3.76). In either case, a = 1172.2. The equation for the demand curve is therefore

$$Q = 1172.2 - 85.7P.$$

b. Now estimate the short-run price elasticity of demand for instant coffee. Derive a linear demand curve for instant coffee.

To find elasticity, first estimate the slope of the demand curve:

$$\frac{\Delta Q}{\Delta P} = \frac{75-70}{10.35-10.48} = \frac{5}{-0.13} = -38.5$$

Given the slope, we can now estimate elasticity using the price and quantity data from the above table. Assuming demand is of the form Q = a – bP, the elasticity will differ in 1997 and 1998, because price and quantity are different. The elasticities at both points and at the average point between the two years are:

$$E_P^{97} = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{10.35}{75}(-38.5) = -5.31$$

$$E_P^{98} = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{10.48}{70}(-38.5) = -5.76$$

$$E_P^{ARC} = \frac{\frac{P_{97}+P_{98}}{2}}{\frac{Q_{97}+Q_{98}}{2}}\frac{\Delta Q}{\Delta P} = \frac{10.415}{72.5}(-38.5) = -5.53$$

To derive the demand curve for instant coffee, note that the slope of the demand curve is –38.5 = –b. To find the coefficient a, use either of the data points from the table above so that a = 75 + 38.5(10.35) = 473.5 or a = 70 + 38.5(10.48) = 473.5. The equation for the demand curve is therefore

$$Q = 473.5 - 38.5P.$$

c. Which coffee has the higher short-run price elasticity of demand? Why do you think this is the case?

Instant coffee is significantly more elastic than roasted coffee. In fact, the demand for roasted coffee is inelastic and the demand for instant coffee is highly elastic. Roasted coffee may have an inelastic demand in the short-run because many people think of coffee as a necessary good. Changes in the price of roasted coffee will not drastically affect the quantity demanded because people want their coffee. Many people, on the other hand, may view instant coffee as a convenient, though imperfect, substitute for roasted coffee.

So, for example, if the price rises a little, the quantity demanded will fall by a large percentage because people would rather drink roasted coffee instead of paying more for a low quality substitute.

PART II
PRODUCERS, CONSUMERS, AND COMPETITIVE MARKETS

CHAPTER 3
CONSUMER BEHAVIOR

TEACHING NOTES

Now we step back from supply and demand analysis to gain a deeper understanding of what lies behind the supply and demand curves. It will help students understand where the course is heading if you explain that this chapter builds the foundation for deriving demand curves in Chapter 4, and that you will do the same for supply curves later in the course (beginning in Chapter 6).

It is important to explain that economists approach behavior somewhat differently than, say, psychologists. We take preferences to be given and don't question how they came to be. Psychologists, on the other hand, are interested in how preferences are formed and how and why they change, among other things. Economists usually assume consumers are rational, while psychologists explore alternative explanations for behavior. It is very useful to describe what we mean by rational, because that term is often misunderstood. We mean that people have goals, and they make decisions that will enable them to achieve those goals. Rational does not mean that the goals are somehow rational or appropriate, nor does it mean that people do what others might think is right or best for them. Economists generally assume that consumers want to maximize their happiness or satisfaction (i.e., utility), and as long as consumers are making decisions that achieve that goal, they are being rational. If someone who absolutely loves fast cars buys an expensive Porsche and consequently lives in a dump, wears worn-out clothes and eats poorly, he is being completely rational if that is what makes him most happy.

Students sometimes think that economists view people as being self-centered and concerned only with themselves. This is not necessarily the case. A consumer's utility can depend on other consumers' purchases or well-being in either a positive or negative way. I had a colleague once who taught Sunday school and said, somewhat jokingly, that it was all just a matter of interdependent utility functions. You can come back to this issue when covering the material on network externalities in Section 4.5 of Chapter 4 if you wish.

Many students find the consumer behavior material to be highly theoretical and not very "realistic." I like to use Milton Friedman's billiards player example to illustrate that theories do not have to be realistic to be useful.[1] In his famous essay, "The Methodology of Positive Economics," Friedman argues that economic theories should be judged by how well they predict and not by the descriptive realism of their assumptions. He suggests that if we wanted to predict how a skilled billiards player would play a particular shot, we could assume that the player knows the laws of physics and can do the calculations in his head to determine how to strike the cue ball. This theory would probably give very good predictions even though the billiards player knows nothing about physics, because he has learned how to make the shot through long practice. Likewise, consumers have learned what makes them happy through experience, so even though the assumption that consumers maximize utility subject to a budget constraint is pretty unrealistic, the theory predicts behavior well and is quite useful.

It is possible to discuss consumer choice without going into extensive detail on utility theory, relying instead on preference relationships and indifference curves. However, if you plan to discuss uncertainty in Chapter 5, you should cover marginal utility (section 3.5). Even if you cover utility theory only briefly, make sure students are comfortable with the term utility because it appears frequently in Chapter 4. Also, emphasize that a consumer's utility for a product does not depend on the price of the product.

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[1] Milton Friedman, "The Methodology of Positive Economics," in *Essays in Positive Economics*, University of Chicago Press, 1953.

Students sometimes say, for example, that they would prefer a huge pickup truck with dual rear wheels to a small convertible, even though they would be much happier driving around in the convertible. The reason they give is that the pickup costs a lot more, so they could sell the pickup, buy the convertible and have money left over to purchase other things. This confuses preference with choice.

When introducing indifference curves, stress that physical quantities are represented on the two axes. After discussing supply and demand, students may think that price should be on the vertical axis. To illustrate indifference curves, pick an initial bundle on the graph and ask which other bundles are likely to be more preferred and less preferred to the initial bundle. This will divide the commodity space into four quadrants as in Figure 3.1, and it is then easier for students to figure out the set of bundles between which the consumer is indifferent. It is helpful to present a lot of examples with different types of goods (and bads) and see if the class can figure out how to draw the indifference curves.

The concept of utility follows naturally from the discussion of indifference curves. Emphasize that it is the ranking that is important and not the utility number, and point out that if we can graph an indifference curve we can certainly find an equation to represent it, so utility functions aren't so far-fetched. Finally, what is most important is the rate at which consumers are willing to exchange goods (the marginal rate of substitution), and this is based on the relative satisfaction that they derive from each good at any particular time.

The marginal rate of substitution, *MRS*, can be confusing. Some students confuse the *MRS* with the ratio of the two quantities. If this is the case, point out that the slope is equal to the ratio of the rise, ΔY, and the run, ΔX. This ratio is equal to the ratio of the intercepts of a line just tangent to the indifference curve. As we move along a convex indifference curve, these intercepts and the *MRS* change. Another problem is the terminology "of *X* for *Y*." This is confusing because we are not substituting "*X* for *Y*," but *Y* for one unit of *X*. You may want to present a variety of examples in class to explain this important concept.

Budget lines are easier to understand than indifference curves for most students, so you need not spend as much time on them. Be certain to point out that the two intercepts represent the number of units of each good the consumer could purchase if the consumer spent all of his or her income on that good. Also be sure to go over some numerical examples illustrating how the budget line shifts with changes in prices and income.

If you want to cover the utility maximization model mathematically, the Appendix to Chapter 4 lays out the Lagrangian method for solving constrained optimization problems and applies it to the maximization of utility subject to a budget constraint. This appendix also shows how demand curves are derived and discusses the Slutsky equation.

QUESTIONS FOR REVIEW

1. What are the four basic assumptions about individual preferences? Explain the significance or meaning of each.

(1) Preferences are complete: this means that the consumer is able to compare and rank all possible baskets of goods and services. (2) Preferences are transitive: this means that preferences are consistent, in that if bundle A is preferred to bundle B and bundle B is preferred to bundle C, then bundle A is preferred to bundle C. (3) More is preferred to less: this means that all goods are desirable, and that the consumer always prefers to have more of a good. (4) Diminishing marginal rate of substitution: this means that indifference curves are convex, and that the slope of the indifference curve increases (becomes less negative) as we move down along the curve.

As a consumer moves down along her indifference curve she is willing to give up fewer units of the good on the vertical axis in exchange for one more unit of the good on the horizontal axis.

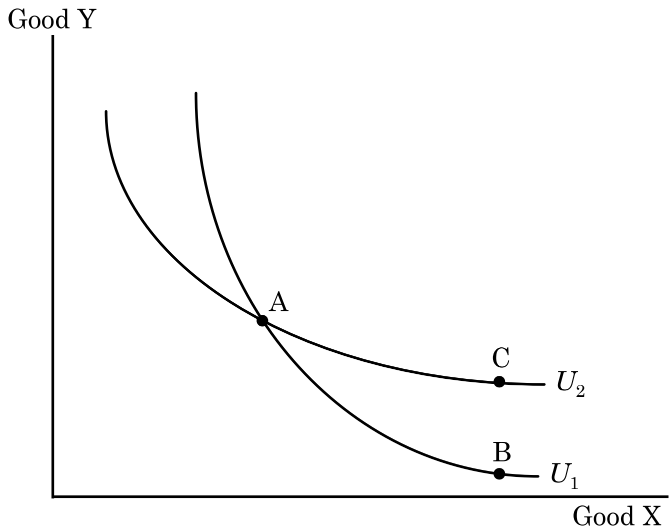
This assumption also means that balanced market baskets are generally preferred to baskets that have a lot of one good and very little of the other good.

2. Can a set of indifference curves be upward sloping? If so, what would this tell you about the two goods?

A set of indifference curves can be upward sloping if we violate assumption number three; more is preferred to less. When a set of indifference curves is upward sloping, it means one of the goods is a "bad" so that the consumer prefers less of that good rather than more. The positive slope means that the consumer will accept more of the bad only if he also receives more of the other good in return. As we move up along the indifference curve the consumer has more of the good he likes, and also more of the good he does not like.

3. Explain why two indifference curves cannot intersect.

The figure below shows two indifference curves intersecting at point *A*. We know from the definition of an indifference curve that the consumer has the same level of utility for every bundle of goods that lies on the given curve. In this case, the consumer is indifferent between bundles *A* and *B* because they both lie on indifference curve U_1. Similarly, the consumer is indifferent between bundles *A* and *C* because they both lie on indifference curve U_2. By the transitivity of preferences this consumer should also be indifferent between *C* and *B*. However, we see from the graph that *C* lies above *B,* so *C* must be preferred to *B* because *C* contains more of Good Y and the same amount of Good X as does *B*, and more is preferred to less. But this violates transitivity, so indifference curves must not intersect.



4. Jon is always willing to trade one can of Coke for one can of Sprite, or one can of Sprite for one can of Coke.

a. What can you say about Jon's marginal rate of substitution?

Jon's marginal rate of substitution can be defined as the number of cans of Coke he would be willing to give up in exchange for a can of Sprite. Since he is always willing to trade one for one, his MRS is equal to 1.

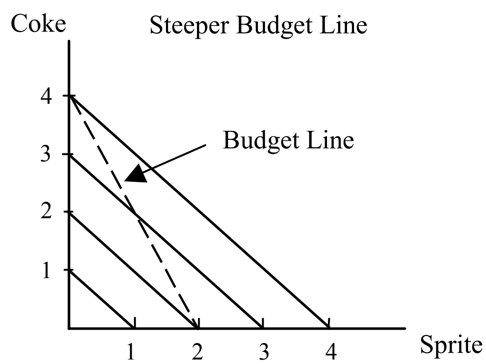
b. Draw a set of indifference curves for Jon.

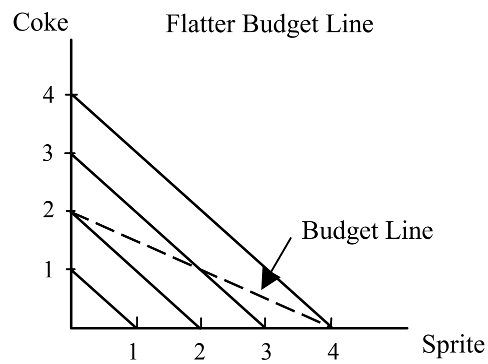
Since Jon is always willing to trade one can of Coke for one can of Sprite, his indifference curves are linear with a slope of –1. See the diagrams below part (c).

c. Draw two budget lines with different slopes and illustrate the satisfaction-maximizing choice. What conclusion can you draw?

Jon's indifference curves are linear with a slope of –1. Jon's budget line is also linear, and will have a slope that reflects the ratio of the two prices. If Jon's budget line is steeper than his indifference curves, he will choose to consume only the good on the vertical axis. If Jon's budget line is flatter than his indifference curves, he will choose to consume only the good on the horizontal axis. Jon will always choose a corner solution where he buys only the less expensive good, unless his budget line has the same slope as his indifference curves. In this case any combination of Sprite and Coke that uses up his entire income will maximize his satisfaction.

The diagrams below show cases where Jon's budget line is steeper than his indifference curves and where it is flatter. Jon's indifference curves are linear with slopes of −1, and four indifference curves are shown in each diagram as solid lines. Jon's budget is \$4.00. In the diagram on the left, Coke costs \$1.00 and Sprite costs \$2.00, so Jon can afford 4 Cokes (if he spends his entire budget on Coke) or 2 Sprites (if he spends his budget on Sprite). His budget line is the dashed line. The highest indifference curve he can reach is the one furthest to the right. He can reach that level of utility by purchasing 4 Cokes and no Sprites. In the diagram on the right, the price of Coke is \$2.00 and the price of Sprite is \$1.00. Jon's budget line is now flatter than his indifference curves, and his optimal bundle is the corner solution with 4 Sprites and no Cokes.





5. What happens to the marginal rate of substitution as you move along a convex indifference curve? A linear indifference curve?

The MRS measures how much of a good you are willing to give up in exchange for one more unit of the other good, keeping utility constant. The MRS diminishes along a convex indifference curve. This occurs because as you move down along the indifference curve, you are willing to give up less and less of the good on the vertical axis in exchange for one more unit of the good on the horizontal axis. The MRS is also the negative of the slope of the indifference curve, which decreases (becomes closer to zero) as you move down along the indifference curve. The MRS is constant along a linear indifference curve because the slope does not change. The consumer is always willing to trade the same number of units of one good in exchange for the other.

6. Explain why an MRS between two goods must equal the ratio of the price of the goods for the consumer to achieve maximum satisfaction.

The MRS describes the rate at which the consumer is willing to trade one good for another to maintain the same level of satisfaction. The ratio of prices describes the trade-off that the consumer is able to make between the same two goods in the market. The tangency of the indifference curve with the budget line represents the point at which the trade-offs are equal and consumer satisfaction is maximized. If the MRS between two goods is not equal to the ratio of prices, then the consumer could trade one good for another at market prices to obtain higher levels of satisfaction. For example, if the slope of the budget line (the ratio of the prices) is –4, the consumer can trade 4 units of Y (the good on the vertical axis) for one unit of X (the good on the horizontal axis). If the MRS at the current bundle is 6, then the consumer is willing to trade 6 units of Y for one unit of X. Since the two slopes are not equal the consumer is not maximizing her satisfaction. The consumer is willing to trade 6 but only has to trade 4, so she should make the trade. This trading continues until the highest level of satisfaction is achieved. As trades are made, the MRS will change and eventually become equal to the price ratio.

7. Describe the indifference curves associated with two goods that are perfect substitutes. What if they are perfect complements?

Two goods are perfect substitutes if the MRS of one for the other is a constant number. In this case, the slopes of the indifference curves are constant, and the indifference curves are therefore linear. If two goods are perfect complements, the indifference curves are L-shaped. In this case the consumer wants to consume the two goods in a fixed proportion, say one unit of good 1 for every one unit of good 2. If she has more of one good but not more of the other then she does not get any extra satisfaction.

8. What is the difference between ordinal utility and cardinal utility? Explain why the assumption of cardinal utility is not needed in order to rank consumer choices.

Ordinal utility implies an ordering among alternatives without regard for intensity of preference. For example, if the consumer's first choice is preferred to his second choice, then utility from the first choice will be higher than utility from the second choice. How much higher is not important. An ordinal utility function generates a ranking of bundles and no meaning is given to the magnitude of the utility number itself. Cardinal utility implies that the intensity of preferences may be quantified, and that the utility number itself has meaning. An ordinal ranking is all that is needed to rank consumer choices. It is not necessary to know how intensely a consumer prefers basket *A* over basket *B*; it is enough to know that *A* is preferred to *B*.

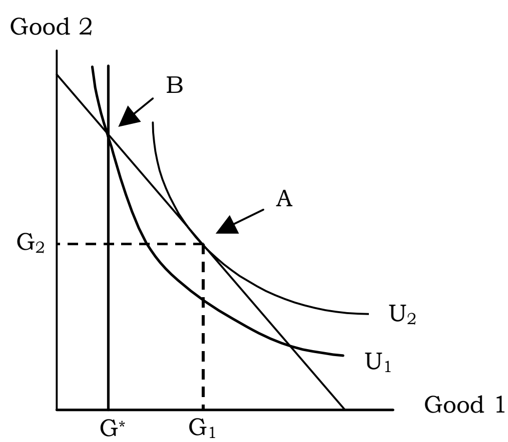
9. Upon merging with the West German economy, East German consumers indicated a preference for Mercedes-Benz automobiles over Volkswagens. However, when they converted their savings into deutsche marks, they flocked to Volkswagen dealerships. How can you explain this apparent paradox?

There is no paradox. Preferences do not involve prices, and East German consumers preferred Mercedes based solely on product characteristics. However, Mercedes prices are considerably higher than Volkswagen prices. So, even though East German consumers preferred a Mercedes to a Volkswagen, they either could not afford a Mercedes or they preferred a bundle of other goods plus a Volkswagen to a Mercedes alone. While the marginal utility of consuming a Mercedes exceeded the marginal utility of consuming a Volkswagen, East German consumers considered the marginal utility per dollar for each good and, for most of them, the marginal utility per dollar was higher for Volkswagens. As a result, they flocked to Volkswagen dealerships to buy VWs.

10. Draw a budget line and then draw an indifference curve to illustrate the satisfaction-maximizing choice associated with two products. Use your graph to answer the following questions.

a. Suppose that one of the products is rationed. Explain why the consumer is likely to be worse off.

When goods are not rationed, the consumer is able to choose the satisfaction-maximizing bundle where the slope of the budget line is equal to the slope of the indifference curve, or the price ratio is equal to the MRS. This is point A in the diagram below where the consumer buys G_1 of good 1 and G_2 of good 2 and achieves utility level U_2. If good 1 is now rationed at G* the consumer will no longer be able to attain the utility maximizing point. He or she cannot purchase amounts of good 1 exceeding G*. As a result, the consumer will have to purchase more of the other good instead. The highest utility level the consumer can achieve with rationing is U_1 at point B. This is not a point of tangency, and the consumer's utility is lower than at point A, so the consumer is worse off as a result of rationing.

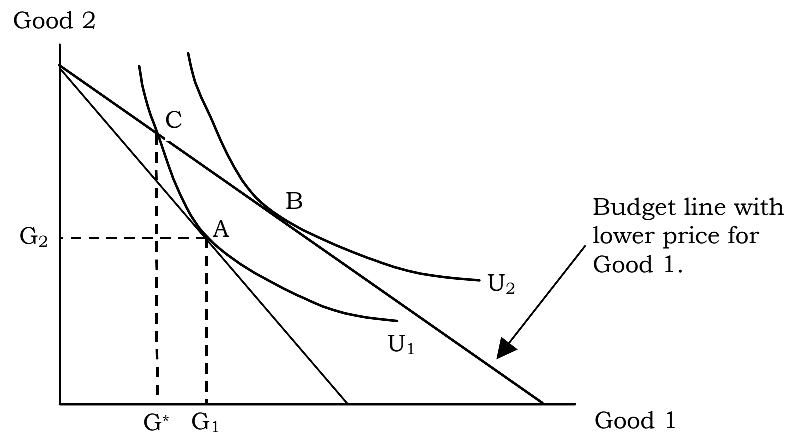


b. Suppose that the price of one of the products is fixed at a level below the current price. As a result, the consumer is not able to purchase as much as she would like. Can you tell if the consumer is better off or worse off?

No, you cannot tell, the consumer could be better off or worse off. When the price of one good is fixed at a level below the current (equilibrium) price, there will be a shortage of that good, and the good will be effectively rationed. In the diagram below, the price of Good 1 has been reduced, and the consumer's budget line has rotated out to the right.

The consumer would like to purchase bundle B, but the amount of Good 1 is restricted because of a shortage. If the most the consumer can purchase is G*, she will be exactly as well off as before, because she will be able to purchase bundle C on her original indifference curve.

If there is more than G* of Good 1 available, the consumer will be better off, and if there is less than G*, the consumer will be worse off.



11. Based on his preferences, Bill is willing to trade 4 movie tickets for 1 ticket to a basketball game. If movie tickets cost $8 each and a ticket to the basketball game costs $40, should Bill make the trade? Why or why not?

No Bill should not make the trade. If he gives up the 4 movie tickets he will save $8 per ticket for a total of $32. However, this is not enough for a basketball ticket, which costs $40. He would have to give up 5 movie tickets to buy a basketball ticket, and he is willing to give up only 4.

12. Describe the equal marginal principle. Explain why this principle may not hold if increasing marginal utility is associated with the consumption of one or both goods.

The equal marginal principle states that to obtain maximum satisfaction the ratio of the marginal utility to price must be equal across all goods. In other words, utility maximization is achieved when the budget is allocated so that the marginal utility per dollar of expenditure (MU/P) is the same for each good. If the MU/P ratios are not equal, allocating more dollars to the good with the higher MU/P will increase utility. As more dollars are allocated to this good its marginal utility will decrease, which causes its MU/P to fall and ultimately equal that of the other goods.

If marginal utility is increasing, however, allocating more dollars to the good with the larger MU/P causes MU to *increase*, and that good's MU/P just keeps getting larger and larger. In this case, the consumer should spend all her income on this good, resulting in a corner solution. With a corner solution, the equal marginal principle *does not* hold.

13. The price of computers has fallen substantially over the past two decades. Use this drop in price to explain why the Consumer Price Index is likely to overstate substantially the cost-of-living index for individuals who use computers intensively.

The consumer price index measures the cost of a basket of goods purchased by a typical consumer in the current year relative to the cost of the basket in the base year. Each good in the basket is assigned a weight, which reflects the importance of the good to the typical consumer, and the weights are kept fixed from year to year.

One problem with fixing the weights is that consumers will shift their purchases from year to year to give more weight to goods whose prices have fallen, and less weight to goods whose prices have risen. The CPI will therefore give too much weight to goods whose prices have risen, and too little weight to goods whose prices have fallen.

In addition, for non-typical individuals who use computers intensively, the fixed weight for computers in the basket will understate the importance of this good, and will hence understate the effect of the fall in the price of computers for these individuals. The CPI will overstate the rise in the cost of living for this type of individual.

14. Explain why the Paasche index will generally understate the ideal cost-of-living index.

The Paasche index measures the current cost of the current bundle of goods relative to the base year cost of the current bundle of goods. The Paasche index will understate the ideal cost of living index because it assumes the individual buys the current year bundle in the base period. In reality, at base year prices the consumer would have been able to attain the same level of utility at a lower cost by altering his or her consumption bundle in light of the base year prices. Since the base year cost is overstated, the denominator of the Paasche index will be too large and the index will be too low, or understated.

EXERCISES

1. In this chapter, consumer preferences for various commodities did not change during the analysis. Yet in some situations, preferences do change as consumption occurs. Discuss why and how preferences might change over time with consumption of these two commodities:

a. cigarettes

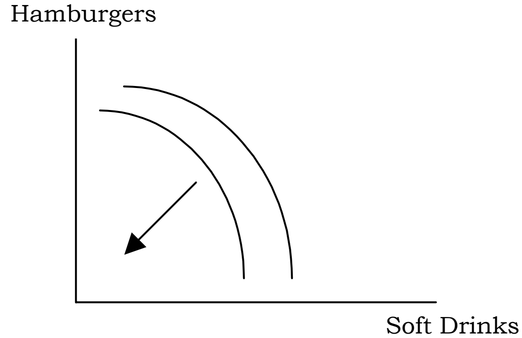
The assumption that preferences do not change is a reasonable one if choices are independent across time. It does not hold, however, when "habit-forming" or addictive behavior is involved, as in the case of cigarettes. The consumption of cigarettes in one period influences the consumer's preference for cigarettes in the next period: the consumer desires cigarettes more because he has become more addicted to them.

b. dinner for the first time at a restaurant with a special cuisine

The first time you eat at a restaurant with a special cuisine can be an exciting new dining experience. This makes eating at the restaurant more desirable. But once you've eaten there, it isn't so exciting to do it again ("been there, done that"), and preference changes. On the other hand, some people prefer to eat at familiar places where they don't have to worry about new and unknown cuisine. For them, the first time at the restaurant would be less pleasant, but once they've eaten there and discovered they like the food, they would find further visits to the restaurant more desirable. In both cases, preferences change as consumption occurs.

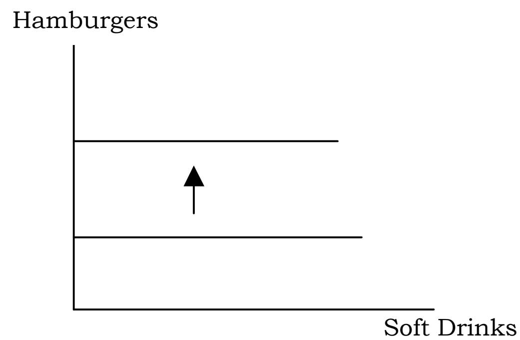
2. Draw indifference curves that represent the following individuals' preferences for hamburgers and soft drinks. Indicate the direction in which the individuals' satisfaction (or utility) is increasing.

a. Joe has convex preferences and dislikes both hamburgers and soft drinks.



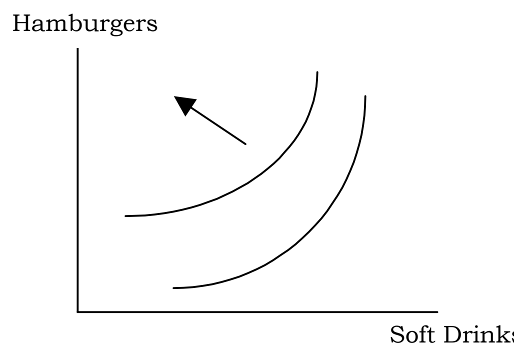
Since Joe dislikes both goods, he prefers less to more, and his satisfaction is increasing in the direction of the origin. Convexity of preferences implies his indifference curves will have the normal shape in that they are bowed towards the direction of increasing satisfaction. Convexity also implies that given any two bundles between which the Joe is indifferent, any linear combination of the two bundles will be in the preferred set, or will leave him at least as well off. This is true of the indifference curves shown in the diagram.

b. Jane loves hamburgers and dislikes soft drinks. If she is served a soft drink, she will pour it down the drain rather than drink it.



Since Jane can freely dispose of the soft drink if it is given to her, she considers it to be a neutral good. This means she does not care about soft drinks one way or the other. With hamburgers on the vertical axis, her indifference curves are horizontal lines. Her satisfaction increases in the upward direction.

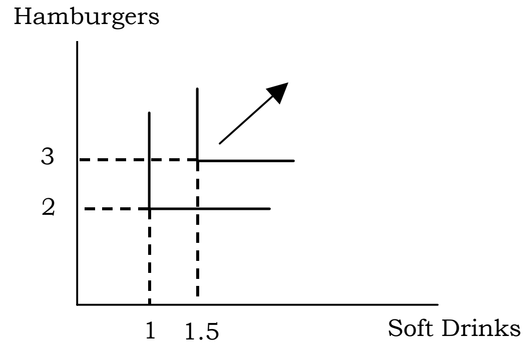
c. Bob loves hamburgers and dislikes soft drinks. If he is served a soft drink, he will drink it to be polite.



Since Bob will drink the soft drink in order to be polite, it can be thought of as a "bad". When served another soft drink, he will require more hamburgers at the same time in order to keep his satisfaction constant. More soft drinks without more hamburgers will worsen his utility. More hamburgers and fewer soft drinks will increase his utility, so his satisfaction increases as we move upward and to the left.

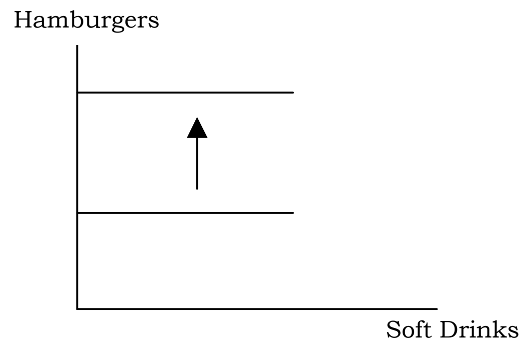
d. Molly loves hamburgers and soft drinks, but insists on consuming exactly one soft drink for every two hamburgers that she eats.

Molly wants to consume the two goods in a fixed proportion so her indifference curves are L-shaped. For a fixed amount of one good, she gets no extra satisfaction from having more of the other good. She will only increase her satisfaction if she has more of both goods.



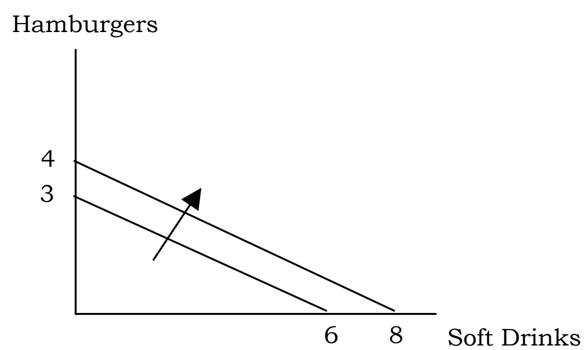
e. Bill likes hamburgers, but neither likes nor dislikes soft drinks.

Like Jane, Bill considers soft drinks to be a neutral good. Since he does not care about soft drinks one way or the other we can assume that no matter how many he has, his utility will be the same. His level of satisfaction depends entirely on how many hamburgers he has, so his satisfaction increases in the upward direction only.



f. Mary always gets twice as much satisfaction from an extra hamburger as she does from an extra soft drink.

How much extra satisfaction Mary gains from an extra hamburger or soft drink tells us something about the marginal utilities of the two goods and about her MRS. If she always receives twice the satisfaction from an extra hamburger, then her marginal utility from consuming an extra hamburger is twice her marginal utility from consuming an extra soft drink. Her MRS, with hamburgers on the vertical axis, is 1/2 because she will give up one hamburger only if she receives two soft drinks. Her indifference curves are straight lines with a slope of −1/2.

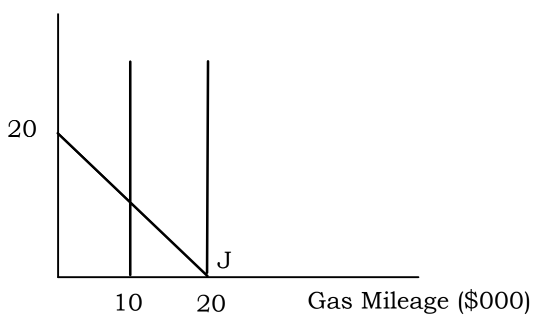


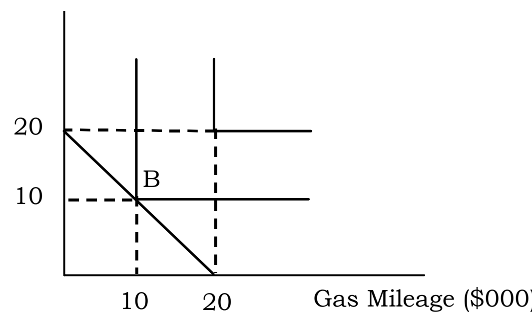
3. If Jane is currently willing to trade 4 movie tickets for 1 basketball ticket, then she must like basketball better than movies. True or false? Explain.

This statement is not necessarily true. If she is *always* willing to trade 4 movie tickets for 1 basketball ticket then yes, she likes basketball better because she will always gain the same satisfaction from 4 movie tickets as she does from 1 basketball ticket. However, it could be that she has convex preferences (diminishing MRS) and is at a bundle where she has a lot of movie tickets relative to basketball tickets. As she gives up movie tickets and acquires more basketball tickets, her MRS will fall. If MRS falls far enough she might get to the point where she would require, say, two basketball tickets to give up another movie ticket.

It would not mean though that she liked basketball better, just that she had a lot of basketball tickets relative to movie tickets. Her willingness to give up a good depends on the quantity of each good in her current basket.

4. Janelle and Brian each plan to spend $20,000 on the styling and gas mileage features of a new car. They can each choose all styling, all gas mileage, or some combination of the two. Janelle does not care at all about styling and wants the best gas mileage possible. Brian likes both equally and wants to spend an equal amount on each. Using indifference curves and budget lines, illustrate the choice that each person will make.



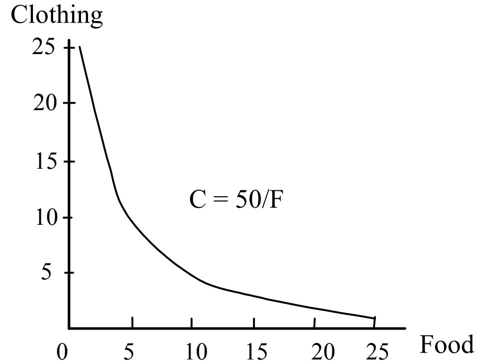


Plot thousands of dollars spent on styling on the vertical axis and thousands spent on gas mileage on the horizontal axis as shown above. Janelle, on the left, has indifference curves that are vertical. If the styling is there she will take it, but she otherwise does not care about it. As her indifference curves move over to the right, she gains more gas mileage and more satisfaction. She will spend all $20,000 on gas mileage at point J. Brian, on the right, has indifference curves that are L-shaped. He will not spend more on one feature than on the other feature. He will spend $10,000 on styling and $10,000 on gas mileage. His optimal bundle is at point B.

5. Suppose that Bridget and Erin spend their incomes on two goods, food (F) and clothing (C). Bridget's preferences are represented by the utility function $U(F,C) = 10FC$, while Erin's preferences are represented by the utility function $U(F,C) = .20F^2C^2$.

a. With food on the horizontal axis and clothing on the vertical axis, identify on a graph the set of points that give Bridget the same level of utility as the bundle (10,5). Do the same for Erin on a separate graph.

The bundle (10,5) contains 10 units of food and 5 of clothing. Bridget receives utility of 10(10)(5) = 500 from this bundle. Thus, her indifference curve is represented by the equation 10FC = 500 or C = 50/F. Some bundles on this indifference curve are (5,10), (10,5), (25,2), and (2,25). It is plotted in the diagram below. Erin receives a utility of $.2(10^2)(5^2)$ = 500 from the bundle (10,5). Her indifference curve is represented by the equation $.2F^2C^2 = 500$, or C = 50/F. This is the same indifference curve as Bridget. Both indifference curves have the normal, convex shape.



b. On the same two graphs, identify the set of bundles that give Bridget and Erin the same level of utility as the bundle (15,8).

For each person, plug F = 15 and C = 8 into their respective utility functions. For Bridget, this gives her a utility of 1200, so her indifference curve is given by the equation 10FC = 1200, or C = 120/F. Some bundles on this indifference curve are (12,10), (10,12), (3,40), and (40,3). The indifference curve will lie above and to the right of the curve diagrammed in part (a). This bundle gives Erin a utility of 2880, so her indifference curve is given by the equation $.2F^2C^2 = 2880$, or C = 120/F. This is the same indifference curve as Bridget.

c. Do you think Bridget and Erin have the same preferences or different preferences? Explain.

They have the same preferences because their indifference curves are identical. This means they will rank all bundles in the same order. Note that it is not necessary that they receive the same level of utility for each bundle to have the same set of preferences. All that is necessary is that they rank the bundles in the same order.

6. Suppose that Jones and Smith have each decided to allocate $1000 per year to an entertainment budget in the form of hockey games or rock concerts. They both like hockey games and rock concerts and will choose to consume positive quantities of both goods. However, they differ substantially in their preferences for these two forms of entertainment. Jones prefers hockey games to rock concerts, while Smith prefers rock concerts to hockey games.

► **Note**: The answer at the end of the book (first printing) is incorrect. In part (a), the labels on the indifference curves should be switched. Jones' indifference curves are more steeply sloped. In part (b), Jones is willing to give up more (not less) of R to get some H than Smith is. So Jones has a higher MRS of H for R (not R for H), and his indifference curves are steeper (not less steep).

a. Draw a set of indifference curves for Jones and a second set for Smith.

Given they each like both goods and they will each choose to consume positive quantities of both goods, we can assume their indifference curves have the normal convex shape. However since Jones has an overall preference for hockey and Smith has an overall preference for rock concerts, their two sets of indifference curves will have different slopes. Suppose that we place rock concerts on the vertical axis and hockey games on the horizontal axis, Jones will have a larger MRS of hockey games for rock concerts than Smith. Jones is willing to give up more rock concerts in exchange for a hockey game since he prefers hockey games. The indifference curves for Jones will therefore be steeper than the indifference curves for Smith.

b. Using the concept of marginal rate of substitution, explain why the two sets of curves are different from each other.

At any combination of hockey games and rock concerts, Jones is willing to give up more rock concerts for an additional hockey game, whereas, Smith is willing to give up fewer rock concerts for an additional hockey game. Since the MRS is a measure of how many of one good (rock concerts) an individual is willing to give up for an additional unit of the other good (hockey games), the MRS, and hence the slope of the indifference curves, will be different for the two individuals.

7. The price of DVDs (D) is $20 and the price of CDs (C) is $10. Philip has a budget of $100 to spend on the two goods. Suppose that he has already bought one DVD and one CD. In addition there are 3 more DVDs and 5 more CDs that he would really like to buy.

a. Given the above prices and income, draw his budget line on a graph with CDs on the horizontal axis.

His budget line is $P_D D + P_C C = I$, or 20D + 10C = 100. If he spends his entire income on DVDs he can afford to buy 5. If he spends his entire income on CDs he can afford to buy 10.

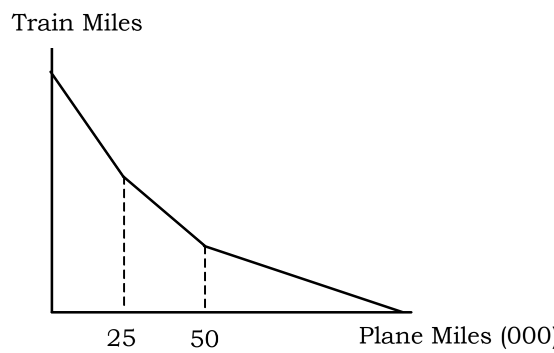
b. Considering what he has already purchased, and what he still wants to purchase, identify the three different bundles of CDs and DVDs that he could choose. For this part of the question, assume that he cannot purchase fractional units.

Given he has already purchased one of each, for a total of $30, he has $70 left. Since he wants 3 more DVDs, he can buy these for $60 and spend his remaining $10 on 1 CD. This is the first bundle below. He could also choose to buy only 2 DVDs for $40 and spend the remaining $30 on 3 CDs. This is the second bundle. Finally, he could purchase 1 more DVD for $20 and spend the remaining $50 on the 5 CDs he would like. This is the final bundle shown in the table below.

Purchased Quantities		**Total Quantities**	
DVDs	CDs	DVDs	CDs
3	1	4	2
2	3	3	4
1	5	2	6

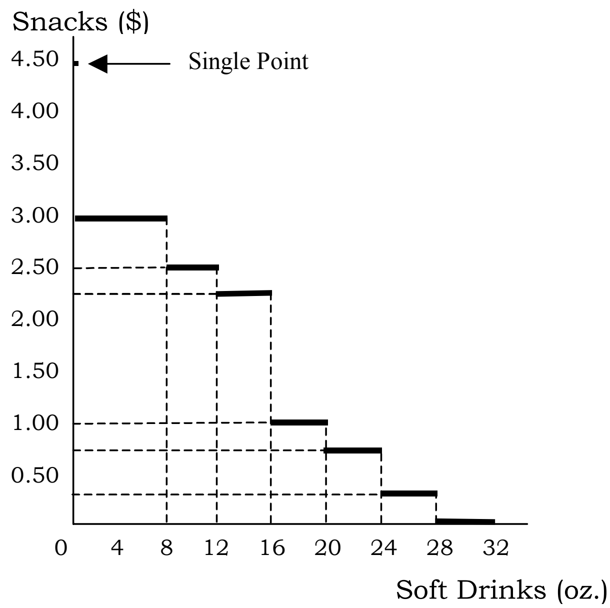
8. Anne has a job that requires her to travel three out of every four weeks. She has an annual travel budget and can travel either by train or by plane. The airline on which she typically flies has a frequent-traveler program that reduces the cost of her tickets according to the number of miles she has flown in a given year. When she reaches 25,000 miles, the airline will reduce the price of her tickets by 25 percent for the remainder of the year. When she reaches 50,000 miles, the airline will reduce the price by 50 percent for the remainder of the year. Graph Anne's budget line, with train miles on the vertical axis and plane miles on the horizontal axis.

The typical budget line is linear (with a constant slope) because the prices of the two goods do not change as the consumer buys more or less of each good. In this case, the price of airline miles changes depending on how many miles Anne purchases. As the price changes, the slope of the budget line changes. Because there are three prices, there will be three slopes (and two kinks) to the budget line. Since the price falls as Anne flies more miles, her budget line will become flatter with every price change.



9. Debra usually buys a soft drink when she goes to a movie theater, where she has a choice of three sizes: the 8-ounce drink costs $1.50, the 12-ounce drink, $2.00, and the 16-ounce drink $2.25. Describe the budget constraint that Debra faces when deciding how many ounces of the drink to purchase. (Assume that Debra can costlessly dispose of any of the soft drink that she does not want.)

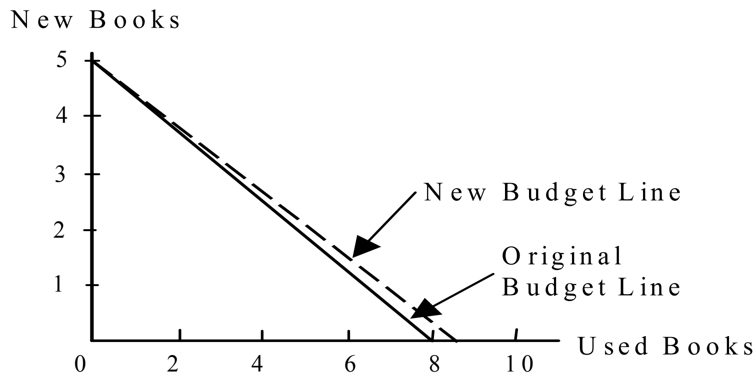
First notice that as the size of the drink increases, the price per ounce decreases. So, for example, if Debra wants 16 ounces of soft drink, she should buy the 16-ounce size and not two 8-ounce size drinks. Also, if Debra wants 14 ounces, she should buy the 16-ounce size drink and dispose of the last 2 ounces. The problem assumes she can do this without cost. As a result, Debra's budget constraint is a series of horizontal lines as shown in the diagram below.



The diagram assumes Debra has a budget of $4.50 to spend on snacks and soft drinks at the movie. Dollars spent on snacks are plotted on the vertical axis and ounces of soft drinks on the horizontal. If Debra wants just an ounce or two of soft drink, she has to purchase the 8-ounce size, which costs $1.50. Thus, she would have $3.00 to spend on snacks. If Debra wants more than 16 ounces of soft drink, she has to purchase more than one drink, and we have to figure out the least-cost way for her to do that. If she wants, say, 20 ounces, she should purchase one 8-ounce and one 12-ounce drink. All of this must be considered in drawing her budget line.

10. Antonio buys five new college textbooks during his first year at school at a cost of $80 each. Used books cost only $50 each. When the bookstore announces that there will be a 10 percent increase in the price of new books and a 5 percent increase in the price of used books, Antonio's father offers him $40 extra.

a. What happens to Antonio's budget line? Illustrate the change with new books on the vertical axis.



In the first year Antonio spends $80 each on 5 new books for a total of $400. For the same amount of money he could have bought 8 used textbooks. His budget line is therefore 80N + 50U = 400, where N is the number of new books and U is the number of used books. After the price change, new books cost $88, used books cost $52.50, and he has an income of $440. If he spends all of his income on new books, he can still afford to buy 5 new books, but he can now afford to buy 8.4 used books if he buys only used books.

The new budget line is 88N + 52.50U = 440. The budget line has become slightly flatter as shown in the diagram.

b. Is Antonio worse or better off after the price change? Explain.

The first year he bought 5 new books at a cost of $80 each, which is a corner solution. The new price of new books is $88 and the cost of 5 new books is now $440. The $40 extra income will cover the price increase. Antonio is definitely not worse off since he can still afford the same number of new books. He may in fact be better off if he decides to switch to some used books, although the slight shift in his budget line suggests that the new optimum will most likely remain at the same corner solution as before.

11. Consumers in Georgia pay twice as much for avocados as they do for peaches. However, avocados and peaches are the same price in California. If consumers in both states maximize utility, will the marginal rate of substitution of peaches for avocados be the same for consumers in both states? If not, which will be higher?

The marginal rate of substitution of peaches for avocados is the maximum amount of avocados that a person is willing to give up to obtain one additional peach, or $MRS = -\frac{\Delta A}{\Delta P}$, where *A* is the number of avocados and *P* the number of peaches. When consumers maximize utility, they set their *MRS* equal to the price ratio, which in this case is $\frac{p_P}{p_A}$, where p_P is the price of a peach and p_A is the price of an avocado. In Georgia, avocados costs twice as much as peaches, so the price ratio is ½, but in California, the prices are the same, so the price ratio is 1. Therefore, when consumers maximize utility (assuming they buy positive amounts of both goods), consumers in Georgia will have a *MRS* that is one-half as large as consumers in California. Thus, the marginal rates of substitution will not be the same for consumers in both states.

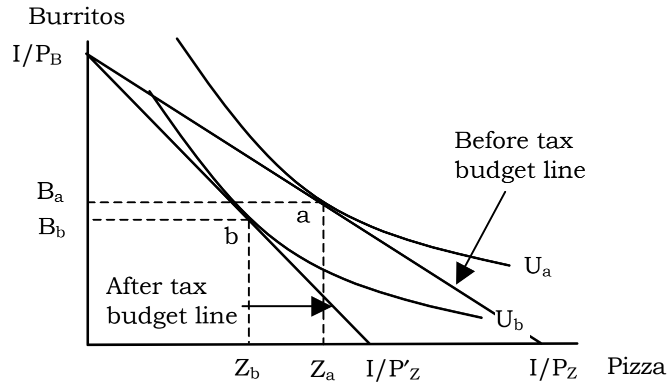
12. Ben allocates his lunch budget between two goods, pizza and burritos.

a. Illustrate Ben's optimal bundle on a graph with pizza on the horizontal axis.

In the diagram below (for part b), Ben's income is I, the price of pizza is P_Z and the price of burritos is P_B. Ben's budget line is linear, and he consumes at the point where his indifference curve is tangent to his budget line at point a in the diagram. This places him on the highest possible indifference curve, which is labeled U_a. Ben buys Z_a pizza and B_a burritos.

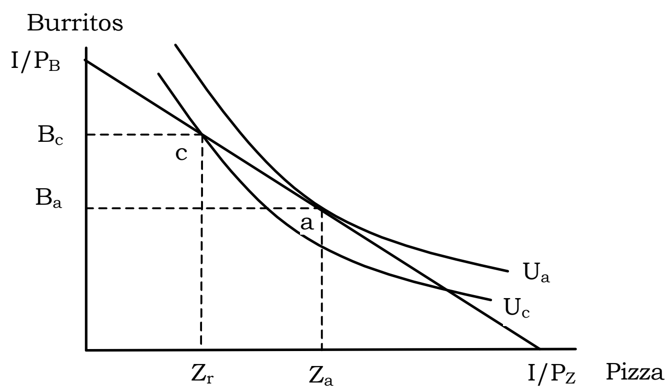
b. Suppose now that pizza is taxed, causing the price to increase by 20 percent. Illustrate Ben's new optimal bundle.

The price of pizza increases 20 percent because of the tax, and Ben's budget line pivots inward. The new price of pizza is $P'_Z = 1.2P_Z$. This shrinks the size of Ben's budget set, and he will no longer be able to afford his old bundle. His new optimal bundle is where the lower indifference curve U_b is tangent to his new budget line. Ben now consumes Z_b pizza and B_b burritos. *Note*: The diagram shows that Ben buys fewer burritos after the tax, but he could buy more if his indifference curves were drawn differently.



c. Suppose instead that pizza is rationed at a quantity less than Ben's desired quantity. Illustrate Ben's new optimal bundle.

Rationing the quantity of pizza that can be purchased will result in Ben not being able to choose his preferred bundle, a. The rationed amount of pizza is Z_r in the diagram. Ben will choose bundle c on the budget line that is above and to the left of his original bundle. He buys more burritos, B_c, and the rationed amount of pizza, Z_r. The new bundle gives him a lower level of utility, U_c.



13. Brenda wants to buy a new car and has a budget of $25,000. She has just found a magazine that assigns each car an index for styling and an index for gas mileage. Each index runs from 1-10, with 10 representing either the most styling or the best gas mileage. While looking at the list of cars, Brenda observes that on average, as the style index increases by one unit, the price of the car increases by $5000. She also observes that as the gas-mileage index rises by one unit, the price of the car increases by $2500.

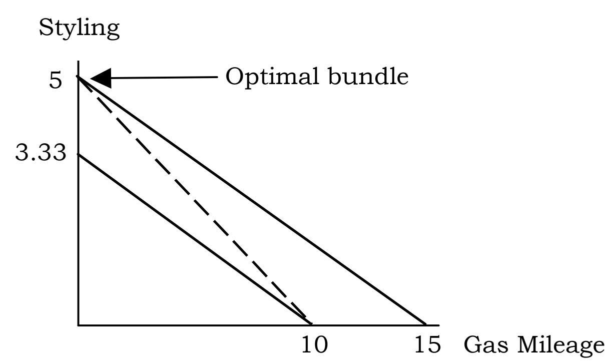
a. Illustrate the various combinations of style (S) and gas mileage (G) that Brenda could select with her $25,000 budget. Place gas mileage on the horizontal axis.

For every $5000 she spends on style the index rises by one so the most she can achieve is a car with a style index of 5. For every $2500 she spends on gas mileage, the index rises by one so the most she can achieve is a car with a gas-mileage index of 10. The slope of her budget line is therefore –1/2 as shown by the dashed line in the diagram for part (b).

b. Suppose Brenda's preferences are such that she always receives three times as much satisfaction from an extra unit of styling as she does from gas mileage. What type of car will Brenda choose?

If Brenda always receives three times as much satisfaction from an extra unit of styling as she does from an extra unit of gas mileage, then she is willing to trade one unit of styling for three units of gas mileage and still maintain the same level of satisfaction. Her indifference curves are straight lines with slopes of –1/3. Two are shown in the graph as solid lines. Since her MRS is a constant 1/3 and the slope of her budget line is –1/2, Brenda will choose all styling.

You can also compute the marginal utility per dollar for styling and gas mileage and note that the MU/P for styling is always greater, so there is a corner solution. Two indifference curves are shown on the graph as solid lines. The higher one starts with styling of 5 on the vertical axis. Moving down the indifference curve, Brenda gives up one unit of styling for every 3 additional units of gas mileage, so this indifference curve intersects the gas mileage axis at 15.



The other indifference curve goes from 3.33 units of styling to 10 of gas mileage. Brenda reaches the highest indifference curve when she chooses all styling and no gas mileage.

c. Suppose that Brenda's marginal rate of substitution (of gas mileage for styling) is equal to S/(4G). What value of each index would she like to have in her car?

To find the optimal value of each index, set MRS equal to the price ratio of 1/2 and cross multiply to get S = 2G. Now substitute into the budget constraint, 5000S + 2500G = 25,000, to get G = 2 and S = 4.

***d.* Suppose that Brenda's marginal rate of substitution (of gas mileage for styling) is equal to (3S)/G. What value of each index would she like to have in her car?**

Now set her new MRS equal to the price ratio of 1/2 and cross multiply to get G = 6S. Now substitute into the budget constraint, 5000S + 2500G = 25,000, to get G = 7.5 and S = 1.25.

14. Connie has a monthly income of $200 that she allocates among two goods: meat and potatoes.

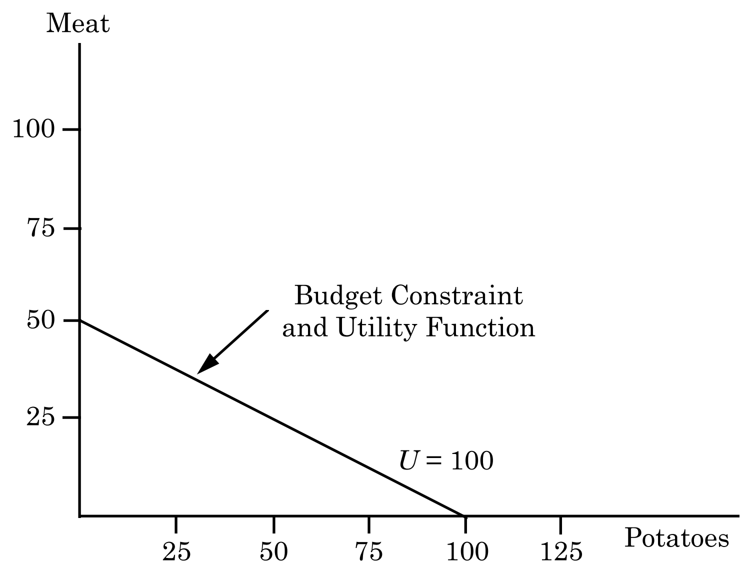
a. Suppose meat costs $4 per pound and potatoes $2 per pound. Draw her budget constraint.

Let M = meat and P = potatoes. Connie's budget constraint is

$$4M + 2P = 200, \text{ or}$$

$$M = 50 - 0.5P.$$

As shown in the figure below, with M on the vertical axis, the vertical intercept is 50 pounds of meat. The horizontal intercept may be found by setting $M = 0$ and solving for P. The horizontal intercept is therefore 100 pounds of potatoes.

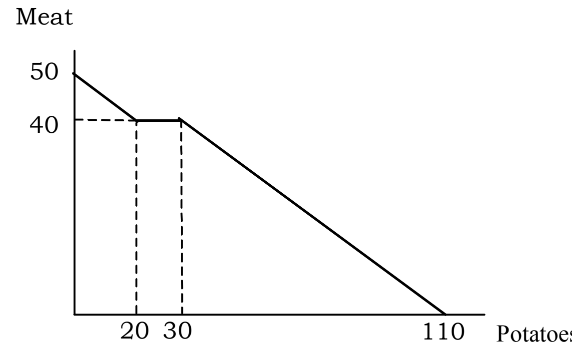


b. Suppose also that her utility function is given by the equation U(M, P) = 2M + P. What combination of meat and potatoes should she buy to maximize her utility? (*Hint*: Meat and potatoes are perfect substitutes.)

When the two goods are perfect substitutes, the indifference curves are linear. To find the slope of the indifference curve, choose a level of utility and find the equation for a representative indifference curve. Suppose U = 50, then 2M + P = 50, or M = 25 – 0.5P. Therefore, Connie's budget line and her indifference curves have the same slope. This indifference curve lies below the one shown in the diagram above. Connie's utility is equal to 100 when she buys 50 pounds of meat and no potatoes or no meat and 100 pounds of potatoes. The indifference curve for U = 100 coincides with her budget constraint. Any combination of meat and potatoes along this line will provide her with maximum utility.

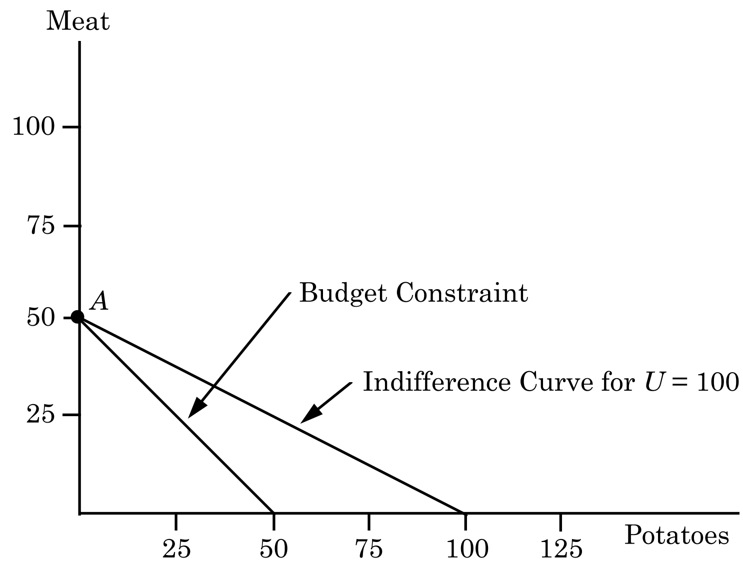
c. Connie's supermarket has a special promotion. If she buys 20 pounds of potatoes (at $2 per pound), she gets the next 10 pounds for free. This offer applies only to the first 20 pounds she buys. All potatoes in excess of the first 20 pounds (excluding bonus potatoes) are still $2 per pound. Draw her budget constraint.

With potatoes on the horizontal axis, Connie's budget constraint has a slope of –1/2 until Connie has purchased twenty pounds of potatoes. Then her budget line is flat from 20 to 30 pounds of potatoes, because the next ten pounds of potatoes are free, and she does not have to give up any meat to get these extra potatoes. After 30 pounds of potatoes, the slope of her budget line becomes –1/2 again until it intercepts the potato axis at 110.



d. An outbreak of potato rot raises the price of potatoes to $4 per pound. The supermarket ends its promotion. What does her budget constraint look like now? What combination of meat and potatoes maximizes her utility?

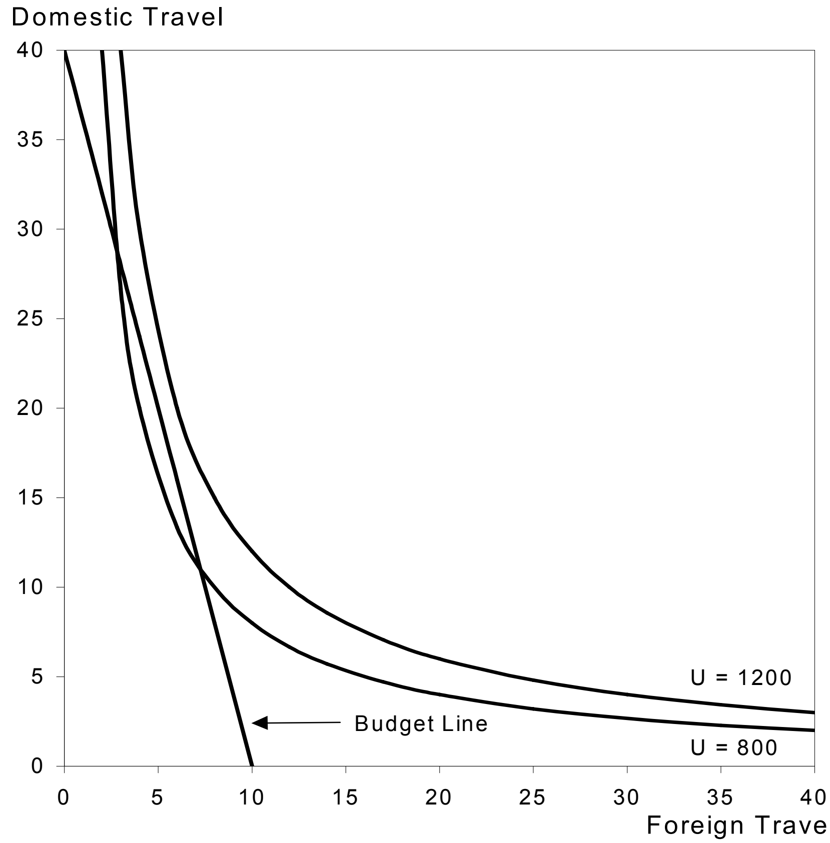
With the price of potatoes at $4, Connie may buy either 50 pounds of meat or 50 pounds of potatoes, or any combination in between. See the diagram below. She maximizes utility at $U = 100$ at point A when she consumes 50 pounds of meat and no potatoes. *This is a corner solution.*



15. Jane receives utility from days spent traveling on vacation domestically (D) and days spent traveling on vacation in a foreign country (F), as given by the utility function U(D,F) = 10DF. In addition, the price of a day spent traveling domestically is $100, the price of a day spent traveling in a foreign country is $400, and Jane's annual travel budget is $4000.

a. Illustrate the indifference curve associated with a utility of 800 and the indifference curve associated with a utility of 1200.

The indifference curve with a utility of 800 has the equation 10DF = 800, or D = 80/F. To plot it, find combinations of D and F that satisfy the equation (such as D = 8 and F = 10). Draw a smooth curve through the points to plot the indifference curve, which is the lower of the two on the graph to the right. The indifference curve with a utility of 1200 has the equation 10DF = 1200, or D = 120/F. Find combinations of D and F that satisfy this equation and plot the indifference curve, which is the upper curve on the graph.



b. Graph Jane's budget line on the same graph.

If Jane spends all of her budget on domestic travel she can afford 40 days. If she spends all of her budget on foreign travel she can afford 10 days. Her budget line is 100D + 400F = 4000, or D = 40 – 4F. This straight line is plotted in the graph above.

c. Can Jane afford any of the bundles that give her a utility of 800? What about a utility of 1200?

Jane can afford some of the bundles that give her a utility of 800 because part of the U = 800 indifference curve lies below the budget line. She cannot afford any of the bundles that give her a utility of 1200 as this indifference curve lies entirely above the budget line.

d. Find Jane's utility maximizing choice of days spent traveling domestically and days spent in a foreign country.

The optimal bundle is where the ratio of prices is equal to the MRS, and Jane is spending her entire income. The ratio of prices is $\frac{P_F}{P_D}=4$, and $MRS=\frac{MU_F}{MU_D}=\frac{10D}{10F}=\frac{D}{F}$. Setting the two equal and solving for D, we get D = 4F.

Substitute this into the budget constraint, 100D + 400F = 4000, and solve for F. The optimal solution is F = 5 and D = 20. Utility is 1000 at the optimal bundle, which is on an indifference curve between the two drawn in the graph above.

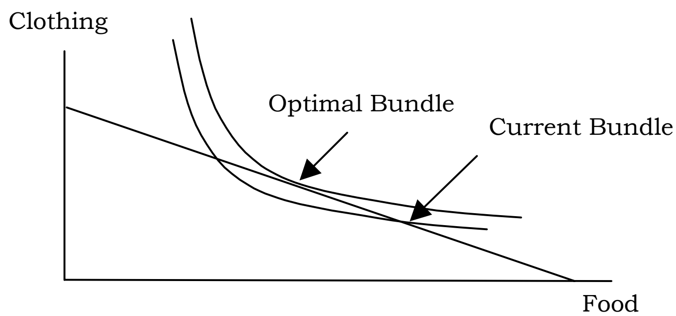
16. Julio receives utility from consuming food (F) and clothing (C) as given by the utility function U(C,F) = FC. In addition, the price of food is $2 per unit, the price of clothing is $10 per unit, and Julio's weekly income is $50.

a. What is Julio's marginal rate of substitution of food for clothing when utility is maximized? Explain.

Plotting clothing on the vertical axis and food on the horizontal, as in the textbook, Julio's utility is maximized when his MRS (of food for clothing) equals P_F/P_C, the price ratio. The price ratio is 2/10 = 0.2, so Julio's MRS will equal 0.2 when his utility is maximized.

b. Suppose instead that Julio is consuming a bundle with more food and less clothing than his utility maximizing bundle. Would his marginal rate of substitution of food for clothing be greater than or less than your answer in part a? Explain.

In absolute value terms, the slope of his indifference curve at this non-optimal bundle is less than the slope of his budget line, because the indifference curve is flatter than the budget line. He is willing to give up more food than he has to at market prices to obtain one more unit of clothing. His MRS is less than the answer in part a.



17. The utility that Meredith receives by consuming food F and clothing C is given by U(F,C) = FC. Suppose that Meredith's income in 1990 is $1200 and that the prices of food and clothing are $1 per unit for each. By 2000, however, the price of food has increased to $2 and the price of clothing to $3. Let 100 represent the cost of living index for 1990. Calculate the ideal and the Laspeyres cost-of-living index for Meredith for 2000. (*Hint*: Meredith will spend equal amounts on food and clothing with these preferences.)

First, we need to calculate F and C, which make up the bundle of food and clothing that maximizes Meredith's utility given 1990 prices and her income in 1990. Use the hint to simplify the problem: Since she spends equal amounts on both goods, she must spend half her income on each. Therefore, $P_F F = P_C C = \$1200/2 = \$600$. Since $P_F = P_C = \$1$, F and C are both equal to 600 units, and Meredith's utility is U = (600)(600) = 360,000.

Note: You can verify the hint mathematically as follows. The marginal utilities with this utility function are $MU_C = \Delta U/\Delta C = F$ and $MU_F = \Delta U/\Delta F = C$. To maximize utility, Meredith chooses a consumption bundle such that $MU_F/MU_C = P_F/P_C$, which yields $P_F F = P_C C$.

Laspeyres Index:

The Laspeyres index represents how much more Meredith would have to spend in 2000 versus 1990 if she consumed the same amounts of food and clothing in 2000 as she did in 1990. That is, the Laspeyres index (LI) for 2000 is given by:

$$LI = 100\,(I')/I,$$

where I′ represents the amount Meredith would spend at 2000 prices consuming the same amount of food and clothing as in 1990. In 2000, 600 clothing and 600 food would cost $3(600) + $2(600) = $3000.

Therefore, the Laspeyres cost-of-living index is:

$$LI = 100(\$3000/\$1200) = 250.$$

Ideal Index:

The ideal index represents how much Meredith would have to spend on food and clothing in 2000 (using 2000 prices) to get the same amount of utility as she had in 1990. That is, the ideal index (II) for 2000 is given by:

$$II = 100(I'')/I, \text{ where } I'' = P'_F F' + P'_C C' = 2F' + 3C',$$

where F′ and C′ are the amount of food and clothing that give Meredith the same utility as she had in 1990. F′ and C′ must also be such that Meredith spends the least amount of money at 2000 prices to attain the 1990 utility level.

The bundle (F′,C′) will be on the same indifference curve as (F,C), so U = F′C′ = FC = 360,000, and 2F′ = 3C′ because Meredith spends the same amount on each good. We now have two equations: F′C′ = 360,000 and 2F′ = 3C′. Solving for F′:

$$F'[(2/3)F'] = 360{,}000 \text{ or } F' = \sqrt{[(3/2)360{,}000)]} = 734.85.$$

From this, we obtain C′,

$$C' = (2/3)F' = (2/3)734.85 = 489.90.$$

In 2000, the bundle of 734.85 units of food and 489.90 units of clothing would cost 734.85($2) + 489.9($3) = $2939.40, and Meredith would still get 360,000 in utility.

We can now calculate the ideal cost-of-living index:

$$II = 100(\$2939.40/\$1200) = 245.$$

CHAPTER 4
INDIVIDUAL AND MARKET DEMAND

TEACHING NOTES

Chapter 4 builds on the consumer choice model presented in Chapter 3. Students find this material very abstract and "unrealistic," so it is important to convince them that there are good reasons for studying how consumers make purchasing decisions in some detail. Most importantly, we gain a deeper understanding of what lies behind demand curves and why, for example, demand curves almost always slope downward. The utility maximizing model is also crucial in determining the supply of labor in Chapter 14, general equilibrium in Chapter 16, and market failure in Chapter 18. So play up these applications when selling students on the importance of the material in this chapter.

Section 4.1 focuses on graphically deriving individual demand and Engel curves by changing price and income. Section 4.2 develops the income and substitution effects of a price change and explains the (perhaps mythical) Giffen good case where the demand curve slopes upward. Section 4.3 shows how to derive the market demand curve from individuals' demand curves. The remaining sections cover consumer surplus, network externalities and empirical demand estimation

When discussing the derivation of demand, review how the budget line pivots around an intercept as price changes and how optimal quantities change as the budget line pivots. Most of the diagrams in the book analyze decreases in price, so you might want to go over an example in which price increases. This will come in handy if you later go over the effects of a gasoline tax in Example 4.2. Once students understand the effect of price changes on consumer choice, they can grasp the derivation of the price-consumption path and the individual demand curve. Remind students that the price a consumer is willing to pay is a measure of the marginal benefit of consuming another unit. This is important for understanding consumer surplus in Section 4.4

Income and substitution effects are difficult for most students, so spend some extra time going over this material. Students frequently have trouble remembering which effect is which on the graph. Emphasize that the substitution effect explains the change in quantity demanded caused by the change in relative prices holding utility constant (a change in the slope of the budget line while staying on the original indifference curve), while the income effect explains the change in quantity demanded caused by a change in purchasing power (a shift of the budget line). Be sure to explain that the substitution effect is always negative (i.e., relative price and quantity are negatively related). On the other hand, the direction of the income effect depends on whether the product is a normal or inferior good. It is a good idea to cover both a price increase and a price decrease.

If students are having trouble understanding the income effect, you might want to give a few numerical examples of how purchasing power changes as the result of a price change. For example, suppose a consumer typically buys a can of soda every day for $.75 per can. If the price increases to $1.00, the consumer's purchasing power drops by $.25, and she will have to spend less on other goods and/or buy less soda. Over a month this amounts to a reduction in real income of roughly $.25 x 30 = $7.50. You can show how the income effect can be large or small depending on how much the consumer spends on the product and how much the price changes.

When covering the aggregation of individual demand curves in Section 4.3, stress that this is equivalent to the horizontal summation of the individual demand curves because we want to add up the quantities demanded by all consumers at each price. To obtain the market demand curve, you must have demand written in the form Q = f(P) as opposed to the inverse demand P = f(Q). The concept of a kink in the market demand curve is often new to students. Emphasize that this is because not all consumers are in the market at all prices. With many consumers there can be many kinks. With thousands of consumers, however, we hardly notice the individual kinks, so it is reasonable to draw most market demand curves as smooth lines.

The concept of elasticity is reintroduced and further explored in Section 4.3. In particular, the relationship between elasticity and revenue is explained.

Here is a way to help students remember this relationship. Think of price and revenue as being connected by a link of some sort. If the link is elastic, price and revenue move in opposite directions because the link between them stretches like a rubber band (for example, an increase in price leads to a decrease in revenue), but if the link is inelastic, price and revenue must move in the same direction because the link is rigid or inflexible.

Consumer surplus is introduced in Section 4.4. Emphasize that it measures the value a consumer places on a good in excess of the price the consumer pays for the good. It is the difference between what the consumer is willing to pay and what he or she actually has to pay for the good. We usually measure consumer surplus using a market demand curve, in which case we are finding the sum of all the individual consumer surpluses. Go over an example with a linear demand curve, and remind students that the area of a triangle is (½)(base)(height). Example 4.5 is a nice application of consumer surplus, but students have a hard time understanding the demand curve for reductions in air pollution, so expect to spend some time if you cover this example. The related topic of producer surplus is covered in Chapter 8, and both producer and consumer surplus are used extensively in Chapter 9 and later chapters

Network externalities in Section 4.5 can be covered quickly, and they are pretty intuitive for most students. Figures 4.16 and 4.17 are a bit complicated, but you do not have to cover them to get the main points across. A nice example of a negative network externality that is covered only briefly in the text is congestion. Road congestion is something most students can relate to, and you could mention the comment about a popular NY City restaurant attributed to Yogi Berra that goes something like, "It's gotten so crowded, nobody goes there anymore."

The first part of Section 4.6, "The Statistical Approach to Demand Estimation," is fairly straightforward, even if you have not covered the forecasting section of Chapter 2. It is important for students to understand that demand curves really do exist and can be estimated. Many seem to think demand curves are figments of economists' imaginations and that we draw them more or less randomly, so this part of the section is very useful. The second part, "The Form of the Demand Relationship," is more complicated and difficult for students who do not understand logarithms.

The Appendix is intended for students with a background in calculus. It goes through the maximization of utility subject to a budget constraint using the Lagrange multiplier method. Demand curves are derived and many of the conditions developed in Chapter 3 are shown mathematically. There is a brief treatment of duality in consumer theory, and the mathematical form for the Slutsky equation is discussed but not derived mathematically.

QUESTIONS FOR REVIEW

1. Explain the difference between each of the following terms:

a. a price consumption curve and a demand curve

The difference between a price consumption curve (PCC) and a demand curve is that the PCC shows the quantities of <u>two goods</u> that a consumer will purchase as the price of one of the goods changes, while a demand curve shows the quantity of <u>one good</u> that a consumer will purchase as the price of that good changes. The graph of the PCC plots the quantity of one good on the horizontal axis and the quantity of the other good on the vertical axis. The demand curve plots the quantity of the good on the horizontal axis and its price on the vertical axis.

b. an individual demand curve and a market demand curve

An individual demand curve plots the quantity demanded by one person at various prices. A market demand curve is the horizontal sum of all the individual demand curves for the product. It plots the total quantity demanded by all consumers at various prices.

c. an Engel curve and a demand curve

An Engel curve shows the quantity of one good that will be purchased by a consumer at different income levels. The quantity of the good is plotted on the horizontal axis and the consumer's income is on the vertical axis. A demand curve is like an Engel curve except that it shows the quantity that will be purchased at different prices instead of different income levels.

d. an income effect and a substitution effect

Both the substitution effect and income effect occur because of a change in the price of a good. The substitution effect is the change in the quantity demanded of the good due to the price change, holding the consumer's utility constant. The income effect is the change in the quantity demanded of the good due to the change in purchasing power brought about by the change in the good's price.

2. Suppose that an individual allocates his or her entire budget between two goods, food and clothing. Can both goods be inferior? Explain.

No, the goods cannot both be inferior; at least one must be a normal good. Here's why. If an individual consumes only food and clothing, then any increase in income must be spent on either food or clothing or both (recall, we assume there are no savings and more of any good is preferred to less, even if the good is an inferior good). If food is an inferior good, then, as income increases, consumption of food falls. With constant prices, the extra income not spent on food must be spent on clothing. Therefore, as income increases, more is spent on clothing, i.e. clothing is a normal good.

3. Explain whether the following statements are true or false.

a. The marginal rate of substitution diminishes as an individual moves downward along the demand curve.

True. The consumer maximizes his utility by choosing the bundle on his budget line where the price ratio is equal to the MRS. For goods 1 and 2, P_1/P_2 = MRS. As the price of good 1 falls, the consumer moves downward along the demand curve for good 1, and the price ratio (P_1/P_2) becomes smaller. Therefore, MRS must also become smaller, and thus MRS diminishes as an individual moves downward along the demand curve.

b. The level of utility increases as an individual moves downward along the demand curve.

True. As the price of a good falls, the budget line pivots outward, and the consumer is able to move to a higher indifference curve.

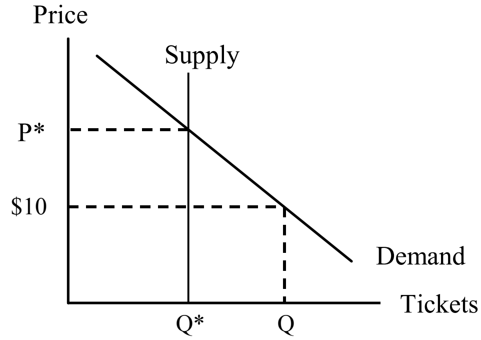
c. Engel curves always slope upwards.

False. If the good is inferior, then as income increases, quantity demanded decreases, and therefore the Engel curve slopes downwards.

4. Tickets to a rock concert sell for $10. But at that price, the demand is substantially greater than the available number of tickets. Is the value or marginal benefit of an additional ticket greater than, less than, or equal to $10? How might you determine that value?

The diagram below shows this situation. At a price of $10, consumers want to purchase Q tickets, but only Q* are available. Consumers would be willing to bid up the ticket price to P*, where the quantity demanded equals the number of tickets available. Since utility-maximizing consumers are willing to pay more than $10, the marginal increase in satisfaction (i.e., the value or marginal benefit of an additional ticket) is greater than $10. One way to determine the value of an additional ticket would be to auction it off.

Another possibility is to allow scalping. Since consumers are willing to pay an amount equal to the marginal benefit they derive from purchasing an additional ticket, the scalper's price equals that value.



5. Which of the following combinations of goods are complements and which are substitutes? Can they be either in different circumstances? Discuss.

a. a mathematics class and an economics class

If the math class and the economics class do not conflict in scheduling, then the classes could be either complements or substitutes. Math is important for understanding economics, and economics can motivate mathematics, so the classes could be complements. If the classes conflict or the student has room for only one in his schedule, they are substitutes.

b. tennis balls and a tennis racket

Tennis balls and a tennis racket are both needed to play tennis, thus they are complements.

c. steak and lobster

Foods can both complement and substitute for each other. Steak and lobster can be substitutes, as when they are listed as separate items on a menu. However, they can also function as complements because they are often served together.

d. a plane trip and a train trip to the same destination

Two modes of transportation between the same two points are substitutes for one another.

e. bacon and eggs

Bacon and eggs are often eaten together and are complementary goods in that case. However, in relation to something else, such as pancakes, bacon and eggs can function as substitutes.

6. Suppose that a consumer spends a fixed amount of income per month on the following pairs of goods:

a. tortilla chips and salsa
b. tortilla chips and potato chips
c. movie tickets and gourmet coffee
d. travel by bus and travel by subway

If the price of one of the goods increases, explain the effect on the quantity demanded of each of the goods. In each pair, which are likely to be complements and which are likely to be substitutes?

a. If the price of tortilla chips increases, the consumer will demand fewer tortilla chips. Since tortilla chips and salsa are complements, the demand curve for salsa will decrease (shift to the left), and the consumer will demand less salsa.

b. If the price of tortilla chips increases, the consumer will demand fewer tortilla chips. Since tortilla chips and potato chips are substitutes, the demand for potato chips will increase (the demand curve will shift to the right), and the consumer will demand more potato chips.

c. The consumer will demand fewer movies after the price increase. You might think the demands for movies and gourmet coffee would be independent of each other. However, because the consumer spends a fixed amount on the two, the demand for coffee will depend on whether the consumer spends more or less of her fixed budget on movies after the price increase. If the consumer's demand elasticity for movie tickets is elastic, she will spend less on movies and, therefore, more of her fixed income will be available to spend on coffee. In this case, her demand for coffee increases, and she buys more gourmet coffee. The goods are substitutes in this situation. If her demand for movies is inelastic, however, she will spend more on movies after the price increase and, therefore, less on coffee. In this case, she will buy less of both goods in response to the price increase for movies, so the goods are complements. Finally, if her demand for movies is unit elastic, she will spend the same amount on movies and therefore will not change her spending on coffee. In this case, the goods are unrelated, and the demand curve for coffee is unchanged.

d. If the price of bus travel increases, the amount of bus travel demanded will fall, and the demand for subway rides will rise, because travel by bus and subway are substitutes. The demand curve for subway rides will shift to the right.

7. Which of the following events would cause a movement *along* the demand curve for U.S. produced clothing, and which would cause a *shift* in the demand curve?

a. the removal of quotas on the importation of foreign clothes

The removal of quotas will allow U.S. consumers to buy more foreign clothing. Because foreign produced goods are substitutes for domestically produced goods, the removal of quotas will result in a decrease in demand (a shift to the left) for U.S. produced clothes. There could be a smaller secondary effect also. When the quotas are removed, the total supply of clothing will increase, causing clothing prices to fall. The drop in clothing prices will lead consumers to buy more U.S. produced clothing, which is a movement along the demand curve.

b. an increase in the income of U.S. citizens

When income rises, expenditures on normal goods such as clothing increase, causing the demand curve to shift out to the right.

c. a cut in the industry's costs of producing domestic clothes that is passed on to the market in the form of lower prices

A cut in an industry's costs will shift the supply curve out. The equilibrium price will fall and quantity demanded will increase. This is a movement along the demand curve.

8. For which of the following goods is a price increase likely to lead to a substantial income (as well as substitution) effect?

a. salt

Small income effect, small substitution effect: The amount of income that is spent on salt is very small, so the income effect is small. Because there are few substitutes for salt, consumers will not readily substitute away from it, and the substitution effect is therefore small.

b. housing

Large income effect, small substitution effect: The amount of income spent on housing is relatively large for most consumers. If the price of housing rises, real income is reduced substantially, leading to a large income effect. However, there are no really close substitutes for housing, so the substitution effect is small.

c. theater tickets

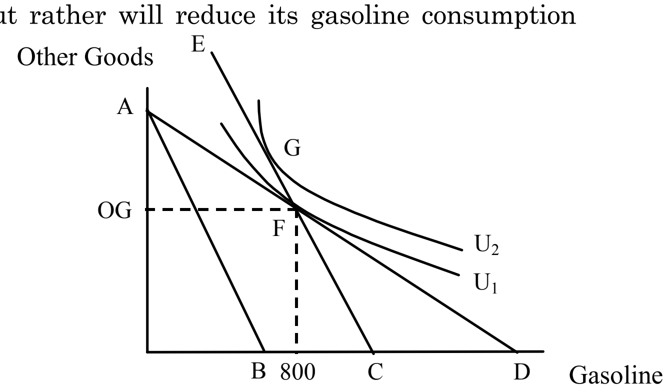
Small income effect, large substitution effect: The amount of income spent on theater tickets is relatively small, so the income effect is small. The substitution effect is large because there are many good substitutes such as movies, TV shows, bowling, dancing and other forms of entertainment.

d. food

Large income effect, virtually no substitution effect: As with housing, the amount of income spent on food is relatively large for most consumers, so the income effect is large. Although consumers can substitute out of particular foods, they cannot substitute out of food in general, so the substitution effect is essentially zero.

9. Suppose that the average household in a state consumes 800 gallons of gasoline per year. A 20-cent gasoline tax is introduced, coupled with a $160 annual tax rebate per household. Will the household be better or worse off under the new program?

If the household does not change its consumption of gasoline, it will be unaffected by the tax-rebate program, because the household pays ($0.20)(800) = $160 in taxes and receives $160 as an annual tax rebate. The two effects cancel each other out. However, the utility maximization model predicts that the household will not continue to purchase 800 gallons of gasoline but rather will reduce its gasoline consumption because of the substitution effect. As a result, it will be better off after the tax and rebate program. The diagram shows this situation. The original budget line is AD, and the household maximizes its utility at point F where the budget line is tangent to indifference curve U_1. At F, the household consumes 800 gallons of gasoline and OG of other goods. The 20-cent increase in price brought about by the tax pivots the budget line to AB (which is exaggerated to make the diagram clearer). Then the $160 rebate shifts the budget line out in a parallel fashion to EC where the household is again able to purchase its original bundle of goods containing 800 gallons of gasoline. However, the new budget line intersects indifference curve U_1 and is not tangent to it. Therefore, point F cannot be the new utility maximizing bundle of goods. The new budget line is tangent to a higher indifference curve U_2 at point G. Point G is therefore the new utility maximizing bundle, and the household consumes less gasoline (because G is to the left of F) and is better off because it is on a higher indifference curve.



10. Which of the following three groups is likely to have the most, and which the least, price-elastic demand for membership in the Association of Business Economists?

a. students

The major differences among the groups are the level of income and commitment to a career in business economics. We know that demand will be more elastic (all else equal) if a good's consumption constitutes a large percentage of an individual's income, because the income effect will be large. Also demand is less elastic the more the good is seen as a necessity. For students, membership in the Association is likely to represent a larger percentage of income than for the other two groups, and students are less likely to see membership as critical for their success. Thus, their demand will be the most elastic.

b. junior executives

The level of income for junior executives will be larger than for students but smaller than for senior executives. They will see membership as important but perhaps not as important as for senior executives. Therefore, their demand will be less elastic than students but more elastic than senior executives.

c. senior executives

The high earnings among senior executives and the high importance they place on membership will result in the least elastic demand for membership.

11. Explain which of the following items in each pair is more price elastic.

a. The demand for a specific brand of toothpaste and the demand for toothpaste in general

The demand for a specific brand is more elastic because the consumer can easily switch to another brand if the price goes up.

b. The demand for gasoline in the short run and the demand for gasoline in the long run

Demand in the long run is more elastic since consumers have more time to adjust to a change in price. For example, consumers can buy more fuel efficient vehicles, move closer to work or school, organize car pools, etc.

12. Explain the difference between a positive and a negative network externality and give an example of each.

A positive network externality exists if one individual's demand increases in response to the purchase of the good by other consumers. Fads are an example of a positive network externality. For example, each individual's demand for baggy pants increases as more other individuals begin to wear baggy pants. This is also called a bandwagon effect. Another example of a positive network externality occurs with communications equipment such as telephones. A telephone is more desirable when there are a large number of other phone owners to whom one can talk. A negative network externality exists if the quantity demanded by one individual decreases in response to the purchase of the good by other consumers. In this case the individual prefers to be different from other individuals. As more people adopt a particular style or purchase a particular type of good, this individual will reduce his demand for the good. Goods like designer clothing can have negative network externalities, as some people would not want to wear the same clothes that many other people are wearing. This is also known as the snob effect. Another example of a negative network externality is road congestion. As more people use a road, the more congested it becomes, and the less valuable it is to each driver.

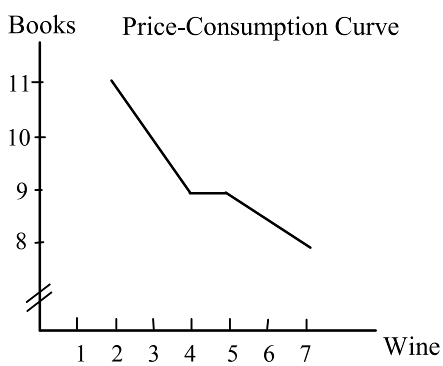
Some people will drive on the road less often (i.e., demand less road services) when it becomes overly congested.

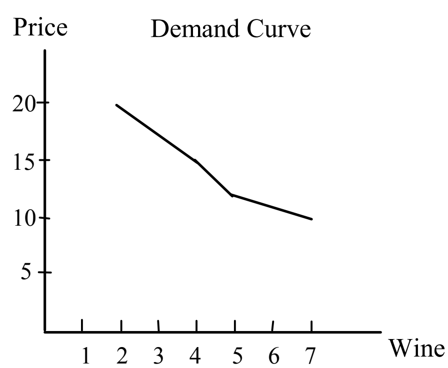
EXERCISES

1. An individual sets aside a certain amount of his income per month to spend on his two hobbies, collecting wine and collecting books. Given the information below, illustrate both the price-consumption curve associated with changes in the price of wine and the demand curve for wine.

Price Wine	Price Book	Quantity Wine	Quantity Book	Budget
$10	$10	7	8	$150
$12	$10	5	9	$150
$15	$10	4	9	$150
$20	$10	2	11	$150

The price-consumption curve connects each of the four optimal bundles given in the table, while the demand curve plots the optimal quantity of wine against the price of wine in each of the four cases. See the diagrams below.

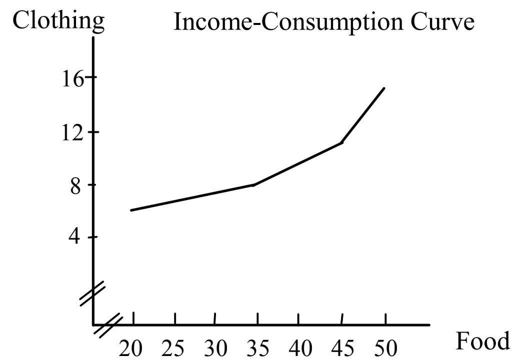


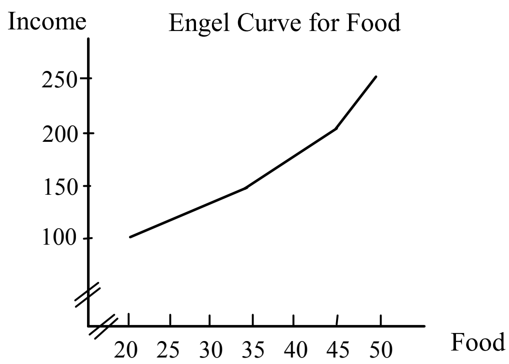


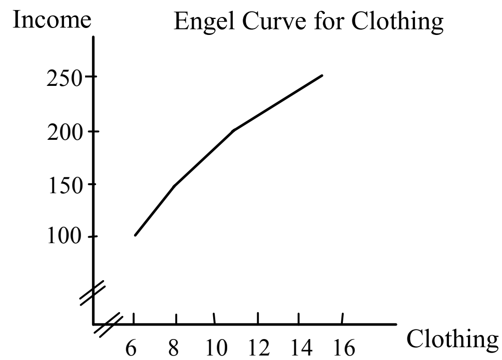
2. An individual consumes two goods, clothing and food. Given the information below, illustrate both the income-consumption curve and the Engel curve for clothing and food.

Price Clothing	Price Food	Quantity Clothing	Quantity Food	Income
$10	$2	6	20	$100
$10	$2	8	35	$150
$10	$2	11	45	$200
$10	$2	15	50	$250

The income-consumption curve (see diagram at right) connects each of the four optimal bundles given in the table above. As the individual's income increases, the budget line shifts out and the optimal bundles change. The Engel curve for each good illustrates the relationship between the quantity consumed and income (on the vertical axis).
Both Engel curves (see diagrams below) are upward sloping, so both goods are normal.







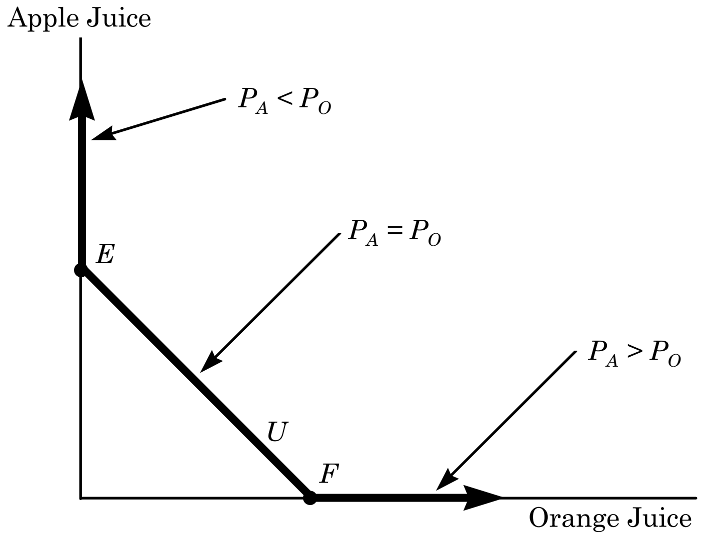
3. Jane always gets twice as much utility from an extra ballet ticket as she does from an extra basketball ticket, regardless of how many tickets of either type she has. Draw Jane's income-consumption curve and her Engel curve for ballet tickets.

Ballet tickets and basketball tickets are perfect substitutes for Jane. Therefore, she will consume either all ballet tickets or all basketball tickets, depending on the two prices. As long as ballet tickets are less than twice the price of basketball tickets, she will choose all ballet. If ballet tickets are more than twice the price of basketball tickets, she will choose all basketball. This can be determined by comparing the marginal utility per dollar for each type of ticket, where her marginal utility from another ballet ticket is 2 times her marginal utility from another basketball ticket regardless of the number of tickets she has. Her income-consumption curve will then lie along the axis of the good that she chooses. As income increases and the budget line shifts out, she will buy more of the chosen good and none of the other good. Her Engel curve for the good chosen is an upward-sloping straight line, with the number of tickets equal to her income divided by the price of the ticket. For the good not chosen, her Engel curve lies on the vertical (income) axis because she will never purchase any of those tickets regardless of how large her income becomes.

4. a. Orange juice and apple juice are known to be perfect substitutes. Draw the appropriate price-consumption curve (for a variable price of orange juice) and income-consumption curve.

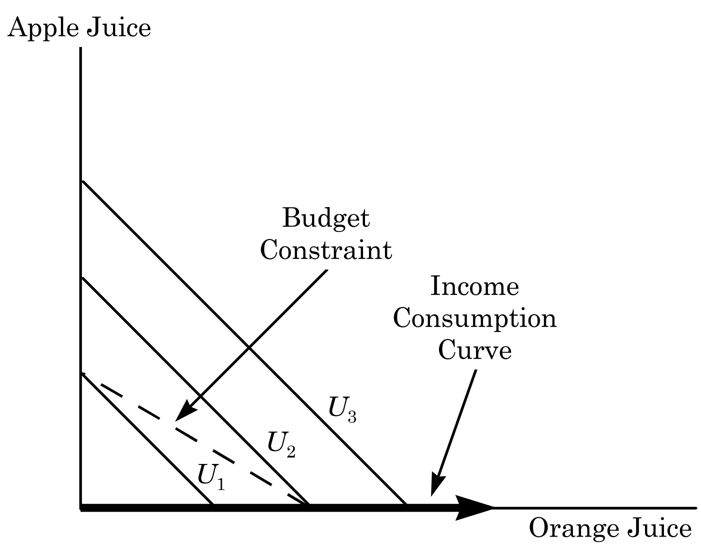
We know that indifference curves for perfect substitutes are straight lines like the line EF in the price-consumption curve diagram below. In this case, the consumer always purchases the cheaper of the two goods (assuming a one-for-one tradeoff).

If the price of orange juice is less than the price of apple juice, the consumer will purchase only orange juice and the price-consumption curve will lie along the orange juice axis of the graph (from point F to the right).



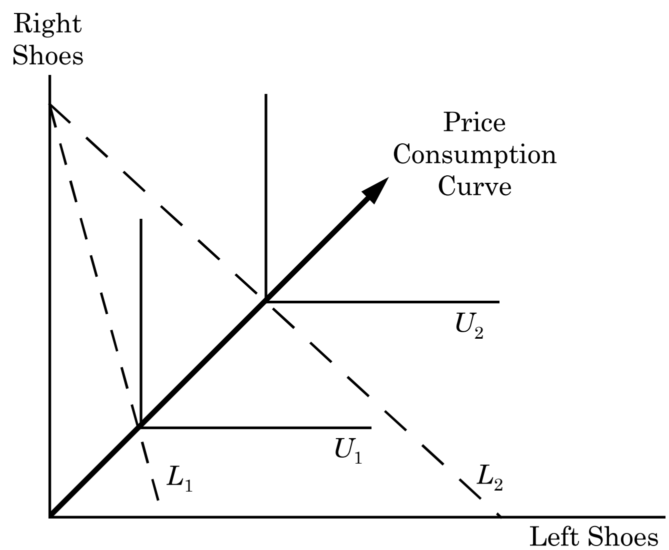
If apple juice is cheaper, the consumer will purchase only apple juice and the price-consumption curve will be on the apple juice axis (above point E). If the two goods have the same price, the consumer will be indifferent between the two; the price-consumption curve will coincide with the indifference curve (between E and F).

Assuming that the price of orange juice is less than the price of apple juice, the consumer will maximize her utility by consuming only orange juice. As income varies, only the amount of orange juice varies. Thus, the income-consumption curve will be the orange juice axis in the figure below. If apple juice were cheaper, the income-consumption curve would lie on the apple juice axis.

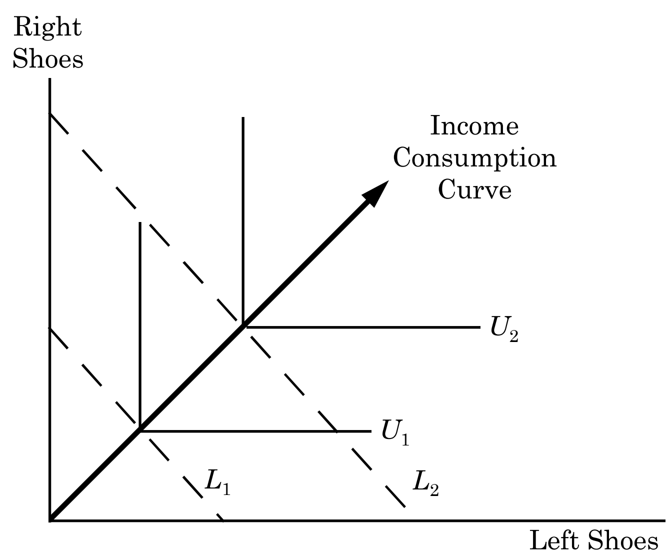


4. b. Left shoes and right shoes are perfect complements. Draw the appropriate price-consumption and income-consumption curves.

For perfect complements, such as right shoes and left shoes, the indifference curves are *L*-shaped. The point of utility maximization occurs when the budget constraints, L_1 and L_2 touch the kink of U_1 and U_2. See the following figure.



In the case of perfect complements, the income consumption curve is also a line through the corners of the *L*-shaped indifference curves. See the figure below.



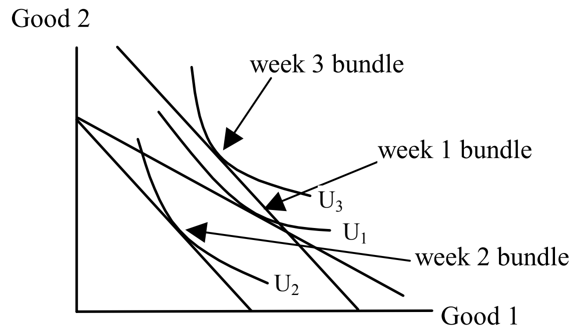
5. Each week, Bill, Mary, and Jane select the quantity of two goods, X_1 and X_2, that they will consume in order to maximize their respective utilities. They each spend their entire weekly income on these two goods.

a. Suppose you are given the following information about the choices that Bill makes over a three-week period:

	x_1	x_2	P_1	P_2	I
Week 1	**10**	**20**	**2**	**1**	**40**
Week 2	**7**	**19**	**3**	**1**	**40**
Week 3	**8**	**31**	**3**	**1**	**55**

Did Bill's utility increase or decrease between week 1 and week 2? Between week 1 and week 3? Explain using a graph to support your answer.

Bill's utility fell between weeks 1 and 2 because he consumed less of both goods in week 2. Between weeks 1 and 2 the price of good 1 rose and his income remained constant. The budget line pivoted inward and he moved from U_1 to a lower indifference curve, U_2, as shown in the diagram. Between week 1 and week 3 his utility rose. The increase in income more than compensated him for the rise in the price of good 1. Since the price of good 1 rose by $1, he would need an extra $10 to afford the same bundle of goods he chose in week 1. This can be found by multiplying week 1 quantities times week 2 prices. However, his income went up by $15, so his budget line shifted out beyond his week 1 bundle. Therefore, his original bundle lies within his new budget set as shown in the diagram, and his new week 3 bundle is on the higher indifference curve U_3.

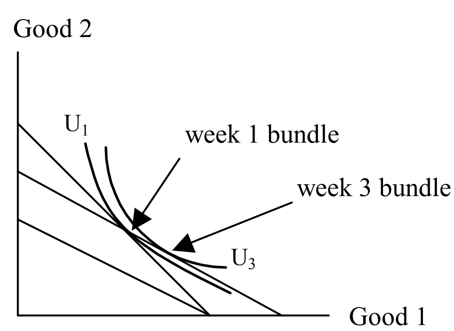


b. Now consider the following information about the choices that Mary makes:

	x_1	x_2	P_1	P_2	I
Week 1	**10**	**20**	**2**	**1**	**40**
Week 2	**6**	**14**	**2**	**2**	**40**
Week 3	**20**	**10**	**2**	**2**	**60**

Did Mary's utility increase or decrease between week 1 and week 3? Does Mary consider both goods to be normal goods? Explain.

Mary's utility went up. To afford the week 1 bundle at the new prices, she would need an extra $20, which is exactly what happened to her income. However, since she could have chosen the original bundle at the new prices and income but did not, she must have found a bundle that left her slightly better off. In the graph to the right, the week 1 bundle is at the point where the week 1 budget line is tangent to indifference curve U_1, which is also the intersection of the week 1 and week 3 budget lines.

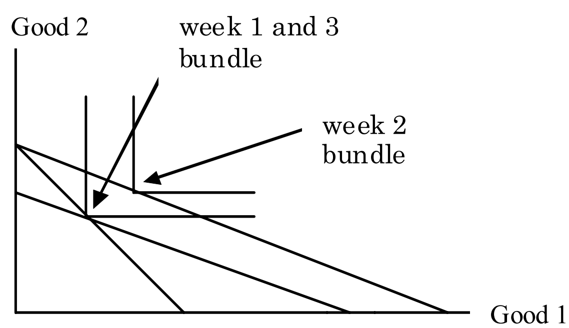


The week 3 bundle is somewhere on the week 3 budget line that lies above the week 1 indifference curve. This bundle will be on a higher indifference curve, U_3 in the graph, and hence Mary's utility increased. A good is normal if more is chosen when income increases. Good 1 is normal because Mary consumed more of it when her income increased (and prices remained constant) between weeks 2 and 3. Good 2 is not normal, however, because when Mary's income increased from week 2 to week 3 (holding prices the same), she consumed less of good 2. Thus good 2 in an inferior good for Mary.

c. **Finally, examine the following information about Jane's choices:**

	x_1	x_2	P_1	P_2	I
Week 1	**12**	**24**	**2**	**1**	**48**
Week 2	**16**	**32**	**1**	**1**	**48**
Week 3	**12**	**24**	**1**	**1**	**36**

Draw a budget line-indifference curve graph that illustrates Jane's three chosen bundles. What can you say about Jane's preferences in this case? Identify the income and substitution effects that result from a change in the price of good X_1.



In week 2, the price of good 1 drops, Jane's budget line pivots outward and she consumes more of both goods. In week 3 the prices remain at the new levels, but Jane's income is reduced. This leads to a parallel leftward shift of her budget line and causes Jane to consume less of both goods. Notice that Jane always consumes the two goods in a fixed 1:2 ratio. This means that Jane views the two goods as perfect complements, and her indifference curves are L-shaped. Intuitively if the two goods are complements, there is no reason to substitute one for the other during a price change, because they have to be consumed in a set ratio. Thus the substitution effect is zero. When the price ratio changes and utility is kept at the same level (as happens between weeks 1 and 3), Jane chooses the same bundle (12, 24), so the substitution effect is zero.

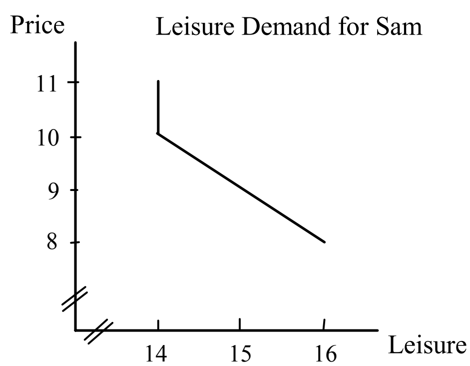
The income effect can be deduced from the changes between weeks 1 and 2 and also between weeks 2 and 3. Between weeks 2 and 3 the only change is the $12 drop in income. This causes Jane to buy 4 fewer units of good 1 and 8 less units of good 2. Because prices did not change, this is purely an income effect. Between weeks 1 and 2, the price of good 1 decreased by $1 and income remained the same. Since Jane bought 12 units of good 1 in week 1, the drop in price increased her purchasing power by ($1)(12) = $12. As a result of this $12 increase in real income, Jane bought 4 more units of good 1 and 8 more of good 2. We know there is no substitution effect, so these changes are due solely to the income effect, which is the same (but in the opposite direction) as we observed between weeks 1 and 2.

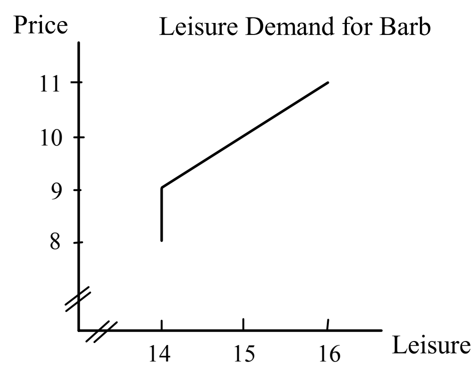
6. Two individuals, Sam and Barb, derive utility from the hours of leisure (L) they consume and from the amount of goods (G) they consume. In order to maximize utility, they need to allocate the 24 hours in the day between leisure hours and work hours. Assume that all hours not spent working are leisure hours. The price of a good is equal to $1 and the price of leisure is equal to the hourly wage. We observe the following information about the choices that the two individuals make:

		Sam	Barb	Sam	Barb
Price of G	**Price of L**	**L (hours)**	**L (hours)**	**G ($)**	**G ($)**
1	8	16	14	64	80
1	9	15	14	81	90
1	10	14	15	100	90
1	11	14	16	110	88

Graphically illustrate Sam's leisure demand curve and Barb's leisure demand curve. Place price on the vertical axis and leisure on the horizontal axis. Given that they both maximize utility, how can you explain the difference in their leisure demand curves?

It is important to remember that less leisure implies more hours spent working. Sam's leisure demand curve is downward sloping. As the price of leisure (the wage) rises, he chooses to consume less leisure and thus spend more time working at a higher wage to buy more goods. Barb's leisure demand curve is upward sloping. As the price of leisure rises, she chooses to consume more leisure (and work less) since her working hours are generating more income per hour. See the leisure demand curves below.





This difference in demand can be explained by examining the income and substitution effects for the two individuals. The substitution effect measures the effect of a change in the price of leisure, keeping utility constant (the budget line rotates along the current indifference curve). Since the substitution effect is always negative, a rise in the price of leisure will cause both individuals to consume less leisure. The income effect measures the effect of the change in purchasing power brought about by the change in the price of leisure. Here, when the price of leisure (the wage) rises, there is an increase in purchasing power (the new budget line shifts outward). Assuming both individuals consider leisure to be a normal good, the increase in purchasing power will increase demand for leisure. For Sam, the reduction in leisure demand caused by the substitution effect outweighs the increase in demand for leisure caused by the income effect, so his leisure demand curve slopes downward. For Barb, her income effect is larger than her substitution effect, so her leisure demand curve slopes upwards.

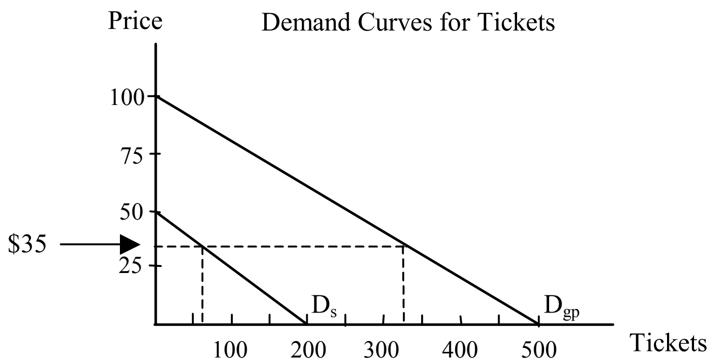
7. The director of a theater company in a small college town is considering changing the way he prices tickets. He has hired an economic consulting firm to estimate the demand for tickets. The firm has classified people who go the theater into two groups, and has come up with two demand functions. The demand curves for the general public (Q_{gp}) and students (Q_s) are given below:

$$Q_{gp} = 500 - 5P$$

$$Q_s = 200 - 4P$$

a. Graph the two demand curves on one graph, with P on the vertical axis and Q on the horizontal axis. If the current price of tickets is $35, identify the quantity demanded by each group.

Both demand curves are downward sloping and linear. For the general public, D_{gp}, the vertical intercept is 100 and the horizontal intercept is 500. For the students, D_s, the vertical intercept is 50 and the horizontal intercept is 200. When the price is $35, the general public demands $Q_{gp} = 500 - 5(35) = 325$ tickets and students demand $Q_s = 200 - 4(35) = 60$ tickets.



b. Find the price elasticity of demand for each group at the current price and quantity.

The elasticity for the general public is $\varepsilon_{gp} = \dfrac{-5(35)}{325} = -0.54$ and the elasticity for students is $\varepsilon_{gp} = \dfrac{-4(35)}{60} = -2.33$. If the price of tickets increases by ten percent then the general public will demand 5.4% fewer tickets and students will demand 23.3% fewer tickets.

c. Is the director maximizing the revenue he collects from ticket sales by charging $35 for each ticket? Explain.

No he is not maximizing revenue because neither of the calculated elasticities is equal to –1. The general public's demand is inelastic at the current price. Thus the director could increase the price for the general public, and the quantity demanded would fall by a smaller percentage, causing revenue to increase. Since the students' demand is elastic at the current price, the director could decrease the price students pay, and their quantity demanded would increase by a larger amount in percentage terms, causing revenue to increase.

d. What price should he charge each group if he wants to maximize revenue collected from ticket sales?

To figure this out, use the formula for elasticity, set it equal to –1, and solve for price and quantity. For the general public:

$$\varepsilon_{gp} = \frac{-5P}{Q} = -1$$
$$5P = Q = 500 - 5P$$
$$P = 50$$
$$Q = 250.$$

For the students:

$$\varepsilon_{s} = \frac{-4P}{Q} = -1$$
$$4P = Q = 200 - 4P$$
$$P = 25$$
$$Q = 100.$$

These prices generate a larger total revenue than the \$35 price. When price is \$35, revenue is $(35)(Q_{gp} + Q_s) = (35)(325 + 60) = \$13,475$. With the separate prices, revenue is $P_{gp}Q_{gp} + P_sQ_s = (50)(250) + (25)(100) = \$15,000$, which is an increase of \$1525, or 11.3%.

8. Judy has decided to allocate exactly \$500 to college textbooks every year, even though she knows that the prices are likely to increase by 5 to 10 percent per year and that she will be getting a substantial monetary gift from her grandparents next year. What is Judy's price elasticity of demand for textbooks? Income elasticity?

Judy will spend the same amount (\$500) on textbooks even when prices increase. We know that total revenue (i.e., total spending on a good) remains constant when price changes only if demand is unit elastic. Therefore Judy's price elasticity of demand for textbooks is –1. Her income elasticity must be zero because she does not plan to purchase more books even though she expects a large monetary gift (i.e., an increase in income).

9. The ACME Corporation determines that at current prices the demand for its computer chips has a price elasticity of –2 in the short run, while the price elasticity for its disk drives is –1.

a. If the corporation decides to raise the price of both products by 10 percent, what will happen to its sales? To its sales revenue?

► **Note**: The answer at the end of the book (first printing) for the percent change in disk drive sales revenue is incorrect. The correct answer is given below.

We know the formula for the elasticity of demand is $E_P = \%\Delta Q/\%\Delta P$. For computer chips, $E_P = -2$, so $-2 = \%\Delta Q/10$, and therefore $\%\Delta Q = -20$. Thus a 10 percent increase in price will reduce the quantity sold by 20 percent. For disk drives, $E_P = -1$, so a 10 percent increase in price will reduce sales by 10 percent.

Sales revenue will decrease for computer chips because demand is elastic and price has increased. We can estimate the change in revenue as follows. Revenue is equal to price times quantity sold. Let $TR_1 = P_1Q_1$ be revenue before the price change and $TR_2 = P_2Q_2$ be revenue after the price change. Therefore $\Delta TR = P_2Q_2 - P_1Q_1$, and thus $\Delta TR = (1.1P_1)(0.8Q_1) - P_1Q_1 = -0.12P_1Q_1$, or a 12 percent decline.

Sales revenue for disk drives will remain unchanged because demand elasticity is –1.

b. Can you tell from the available information which product will generate the most revenue? If yes, why? If not, what additional information do you need?

No. Although we know the elasticities of demand, we do not know the prices or quantities sold, so we cannot calculate the revenue for either product. We need to know the prices of chips and disk drives and how many of each ACME sells.

10. By observing an individual's behavior in the situations outlined below, determine the relevant income elasticities of demand for each good (i.e., whether the good is normal or inferior). If you cannot determine the income elasticity, what additional information do you need?

a. Bill spends all his income on books and coffee. He finds $20 while rummaging through a used paperback bin at the bookstore. He immediately buys a new hardcover book of poetry.

Books are a normal good since his consumption of books increases with income. Coffee is a neutral good since consumption of coffee stayed the same when income increased.

b. Bill loses $10 he was going to use to buy a double espresso. He decides to sell his new book at a discount to a friend and use the money to buy coffee.

When Bill's income decreased by $10 he decided to own fewer books, so books are a normal good. Coffee appears to be a neutral good because Bill's purchase of the double espresso did not change as his income changed.

c. Being bohemian becomes the latest teen fad. As a result, coffee and book prices rise by 25 percent. Bill lowers his consumption of both goods by the same percentage.

Books and coffee are both normal goods because Bill's response to a decline in real income is to decrease consumption of both goods. In addition, the income elasticities for both goods are the same because Bill reduces consumption of both by the same percentage.

d. Bill drops out of art school and gets an M.B.A. instead. He stops reading books and drinking coffee. Now he reads *The Wall Street Journal* and drinks bottled mineral water.

His tastes have changed completely, and we do not know how he would respond to price and income changes. We need to observe how his consumption of the *WSJ* and bottled water changes as his income changes.

11. Suppose the income elasticity of demand for food is 0.5 and the price elasticity of demand is –1.0. Suppose also that Felicia spends $10,000 a year on food, the price of food is $2, and that her income is $25,000.

a. If a sales tax on food caused the price of food to increase to $2.50, what would happen to her consumption of food? (*Hint*: Since a large price change is involved, you should assume that the price elasticity measures an arc elasticity, rather than a point elasticity.)

The arc elasticity formula is:

$$E_P = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{(P_1 + P_2)/2}{(Q_1 + Q_2)/2}\right).$$

We know that $E_P = -1$, $P_1 = 2$, $P_2 = 2.50$ (so $\Delta P = 0.50$), and $Q_1 = 5000$ units (because Felicia spends \$10,000 and each unit of food costs \$2). We also know that Q_2, the new quantity, is $Q_2 = Q_1 + \Delta Q$. Thus, if there is no change in income, we may solve for ΔQ:

$$-1 = \left(\frac{\Delta Q}{0.5}\right)\left(\frac{(2 + 2.5)/2}{(5000 + (5000 + \Delta Q))/2}\right).$$

By cross-multiplying and rearranging terms, we find that $\Delta Q = -1000$. This means that she decreases her consumption of food from 5000 to 4000 units. As a check, recall that total spending should remain the same because the price elasticity is –1. After the price change, Felicia spends (\$2.50)(4000) = \$10,000, which is the same as she spent before the price change.

b. Suppose that Felicia gets a tax rebate of \$2500 to ease the effect of the sales tax. What would her consumption of food be now?

A tax rebate of \$2500 is an income increase of \$2500. To calculate the response of demand to the tax rebate, use the definition of the arc elasticity of income.

$$E_I = \left(\frac{\Delta Q}{\Delta I}\right)\left(\frac{(I_1 + I_2)/2}{(Q_1 + Q_2)/2}\right).$$

We know that $E_I = 0.5$, $I_1 = 25{,}000$, $\Delta I = 2500$ (so $I_2 = 27{,}500$), and $Q_1 = 4000$ (from the answer to 11a). Assuming no change in price, we solve for ΔQ.

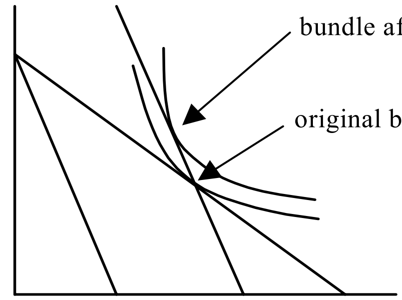
$$0.5 = \left(\frac{\Delta Q}{2500}\right)\left(\frac{(25{,}000 + 27{,}500)/2}{(4000 + (4000 + \Delta Q))/2}\right).$$

By cross-multiplying and rearranging terms, we find that $\Delta Q = 195$ (approximately). This means that she increases her consumption of food from 4000 to 4195 units.

c. Is she better or worse off when given a rebate equal to the sales tax payments? Draw a graph and explain.

► **Note**: The answer at the end of the book (first printing) used incorrect quantities and prices. The correct answer is given below.

Felicia is better off after the rebate. The amount of the rebate is enough to allow her to purchase her original bundle of food and other goods. Recall that originally she consumed 5000 units of food. When the price went up by fifty cents per unit, she needed an extra (5000)(\$0.50) = \$2500 to afford the same quantity of food without reducing the quantity of the other goods consumed. This is the exact amount of the rebate. However, she did not choose to return to her original bundle. We can therefore infer that she found a better bundle that gave her a higher level of utility. In the graph below, when the price of food increases, the budget line pivots inward. When the rebate is given, this new budget line shifts out to the right in a parallel fashion. The bundle after the rebate is on that part of the new budget line that was previously unaffordable, and that lies above the original indifference curve. It is on a higher indifference curve, so Felicia is better off after the rebate.



12. You run a small business and would like to predict what will happen to the quantity demanded for your product if you raise your price. While you do not know the exact demand curve for your product, you do know that in the first year you charged $45 and sold 1200 units and that in the second year you charged $30 and sold 1800 units.

a. If you plan to raise your price by 10 percent, what would be a reasonable estimate of what will happen to quantity demanded in percentage terms?

We must first find the price elasticity of demand. Because the price and quantity changes are large in percentage terms, it is best to use the arc elasticity measure. $E_P = (\Delta Q/\Delta P)(\text{average } P/\text{average } Q) = (600/-15)(37.50/1500) = -1$. With an elasticity of –1, a 10 percent increase in price will lead to a 10 percent decrease in quantity.

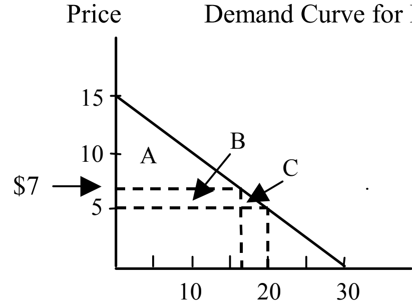
b. If you raise your price by 10 percent, will revenue increase or decrease?

When elasticity is –1, revenue will remain constant if price is increased.

13. Suppose you are in charge of a toll bridge that costs essentially nothing to operate. The demand for bridge crossings *Q* is given by $P = 15 - \frac{1}{2}Q$.

a. Draw the demand curve for bridge crossings.

The demand curve is linear and downward sloping. The vertical intercept is 15 and the horizontal intercept is 30.



b. How many people would cross the bridge if there were no toll?

At a price of zero, 0 = 15 – (1/2)Q, so Q = 30. The quantity demanded would be 30.

c. What is the loss of consumer surplus associated with a bridge toll of $5?

If the toll is $5 then the quantity demanded is 20. The lost consumer surplus is the difference between the consumer surplus when price is zero and the consumer surplus when price is $5. When the toll is zero, consumer surplus is the entire area under the demand curve, which is (1/2)(30)(15) = 225. When P = 5, consumer surplus is area A + B + C in the graph above. The base of this triangle is 20 and the height is 10, so consumer surplus = (1/2)(20)(10) = 100. The loss of consumer surplus is therefore 225 – 100 = $125.

d. The toll-bridge operator is considering an increase in the toll to $7. At this higher price, how many people would cross the bridge? Would the toll-bridge revenue increase or decrease? What does your answer tell you about the elasticity of demand?

At a toll of $7, the quantity demanded would be 16. The initial toll revenue was $5(20) = $100. The new toll revenue is $7(16) = $112. Since the revenue went up when the toll was increased, demand is inelastic (the 40% increase in price outweighed the 20% decline in quantity demanded).

e. Find the lost consumer surplus associated with the increase in the price of the toll from $5 to $7.

The lost consumer surplus is area B + C in the graph above. Thus, the loss in consumer surplus is (16)(7 – 5) + (1/2)(20 – 16)(7 – 5) = $36.

14. Vera has decided to upgrade the operating system on her new PC. She hears that the new Linux operating system is technologically superior to Windows and substantially lower in price. However, when she asks her friends, it turns out they all use PCs with Windows. They agree that Linux is more appealing but add that they see relatively few copies of Linux on sale at local stores. Vera chooses Windows. Can you explain her decision?

Vera is influenced by a positive network externality (not a bandwagon effect). When she hears that there are limited software choices that are compatible with Linux and that none of her friends use Linux, she decides to go with Windows. If she had not been interested in acquiring much software and did not think she would need to get advice from her friends, she might have purchased Linux.

15. Suppose that you are the consultant to an agricultural cooperative that is deciding whether members should cut their production of cotton in half next year. The cooperative wants your advice as to whether this action will increase members' revenues. Knowing that cotton (C) and watermelons (W) both compete for agricultural land in the South, you estimate the demand for cotton to be $C = 3.5 - 1.0P_C + 0.25P_W + 0.50I$, where P_C is the price of cotton, P_W the price of watermelon, and I income. Should you support or oppose the plan? Is there any additional information that would help you to provide a definitive answer?

If production of cotton is cut in half, then the price of cotton will increase, given that we see from the equation above that demand is downward sloping. With price increasing and quantity demanded decreasing, revenue could go either way. It depends on whether demand is elastic or inelastic. If demand is elastic, a decrease in production and an increase in price would decrease revenue. If demand is inelastic, a decrease in production and an increase in price would increase revenue. You need a lot of information before you can give a definitive answer. First, you must know the current prices for cotton and watermelon plus the level of income; then you can calculate the quantity of cotton demanded, C. Next, you have to cut C in half and determine the effect that will have on the price of cotton, assuming that income and the price of watermelons are not affected (which is a big assumption). Then you can calculate the original revenue and the new revenue to see whether this action increases members' revenues or not.

CHAPTER 4 APPENDIX
DEMAND THEORY – A MATHEMATICAL TREATMENT

EXERCISES

1. Which of the following utility functions are consistent with convex indifference curves and which are not?

a. U(X, Y) = 2X + 5Y

b. U(X, Y) = $(XY)^{0.5}$

c. U(X, Y) = Min(X, Y), where Min is the minimum of the two values of X and Y.

Indifference maps for the three utility functions are presented in Figures 4A.1(a), 4A.1(b), and 4A.1(c). The first is a series of straight lines, the second is a series of hyperbolas and the third is a series of *L*-shaped curves. Only the second utility function has strictly convex indifference curves.

To graph the indifference curves which represent the preferences given by U(X,Y) = 2X + 5Y, set utility equal to some level, U_0, and solve for Y to get

$$Y = \frac{U_0}{5} - \frac{2}{5}X .$$

Since this is the equation for a straight line, the indifference curves are linear with intercept $\frac{U_0}{5}$ and slope $-\frac{2}{5}$. The graph shows three indifference curves for three different values of U, where $U_0 < U_1 < U_2$.

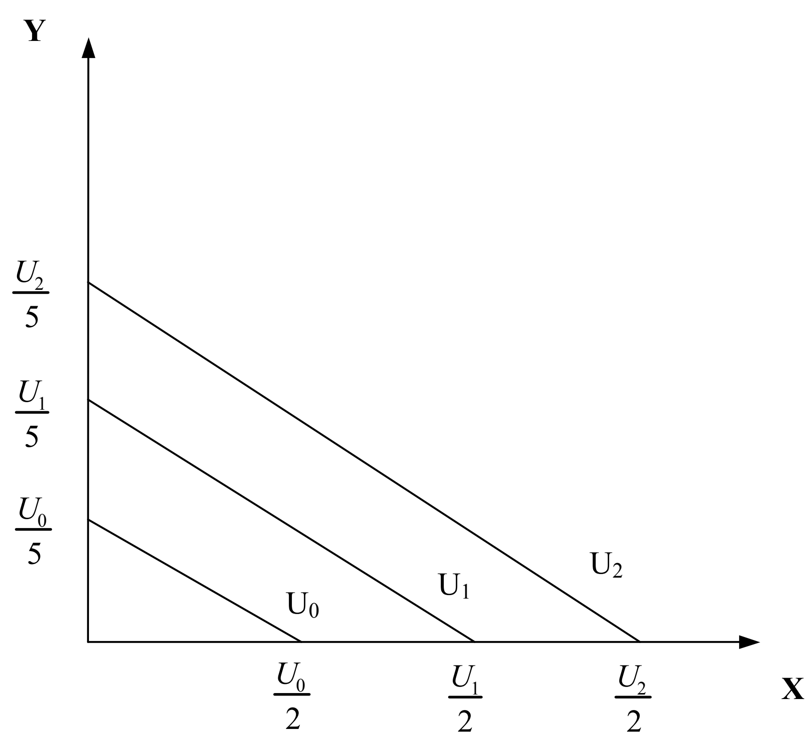


Figure 4A.1(a)

To graph the indifference curves that represent the preferences given by $U(X,Y) = (XY)^{0.5}$, set utility equal a given level U_0 and solve for Y to get

$$Y = \frac{U_0^2}{X}.$$

By plugging in a few values for X and solving for Y, you will be able to graph the indifference curve for utility value U_0, which is illustrated in Figure 4A.1(b), along with the indifference curve for a larger utility value, U_1.

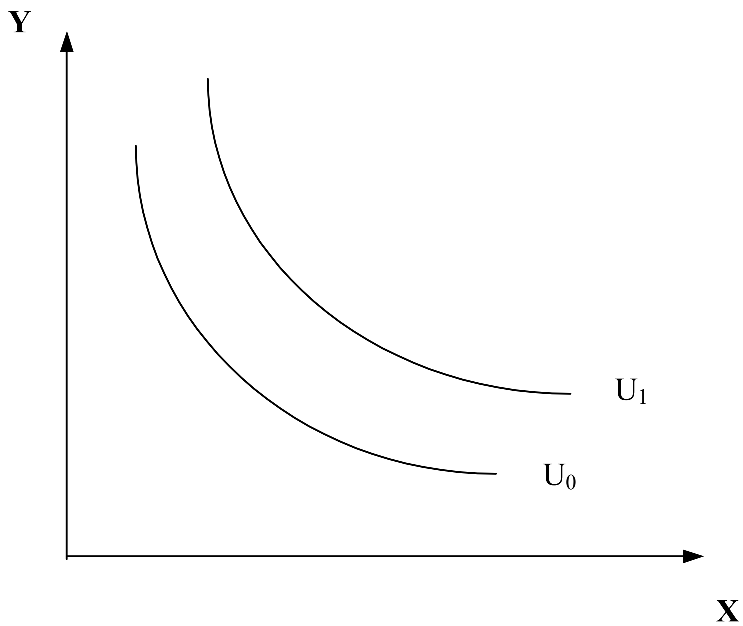


Figure 4A.1(b)

To graph the indifference curves which represent the preferences given by $U(X,Y) = Min(X,Y)$, first note that utility functions of this form result in indifference curves that are L-shaped and represent a complementary relationship between X and Y. In this case, for any given level of utility U_0, the minimum value of X and Y will also be equal to U_0. If X increases but Y does not, utility will not change. If both X and Y change, then utility will change, and we will move to a different indifference curve. See the following table which illustrates how the utility value depends on the amounts of *X* and *Y* in the consumption bundle.

X	Y	U
10	10	10
10	12	10
12	12	12
12	11	11
8	11	8
8	9	8

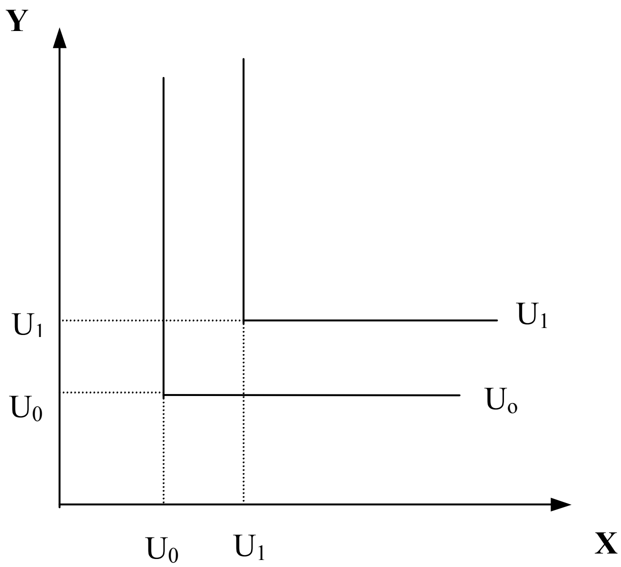


Figure 4A.1(c)

2. Show that the two utility functions given below generate the identical demand functions for goods X and Y:

a. U(X, Y) = log(X) + log(Y)

b. U(X, Y) = $(XY)^{0.5}$

If two utility functions are equivalent, then the demand functions derived from them are identical. Two utility functions are equivalent if you can transform one of them and get the other one. The transformation must be performed by a function that transforms one set of numbers into another set without changing their order. So, for example, the square function could be used, because it does not change the order of numbers that are squared. If w is larger than z, then w^2 is larger than z^2. The logarithm function can also be used as a transformation function, and that is what we use here.

Taking the logarithm of $U(X, Y) = (XY)^{0.5}$ we obtain

$$\log U(X, Y) = 0.5 \log(XY) = 0.5 (\log(X) + \log(Y)).$$

Now multiply both sides by 2, which yields the utility function in a.

$$2[\log U(X,Y)] = \log(X) + \log(Y).$$

Therefore, the two utility functions are equivalent and will yield identical demand functions. We can also demonstrate this directly by solving for the demand functions in both cases and showing that they are the same.

a. To find the demand functions for X and Y, corresponding to $U(X, Y) = \log(X) + \log(Y)$, we must maximize $U(X, Y)$ subject to the budget constraint. To do this, first write out the Lagrangian function, where λ is the Lagrange multiplier:

$$\Phi = \log(X) + \log(Y) - \lambda(P_X X + P_Y Y - I).$$

Differentiating with respect to X, Y and λ, and setting the derivatives equal to zero:

$$\frac{\partial\Phi}{\partial X} = \frac{1}{X} - \lambda P_X = 0$$

$$\frac{\partial\Phi}{\partial Y} = \frac{1}{Y} - \lambda P_Y = 0$$

$$\frac{\partial\Phi}{\partial\lambda} = I - P_X X - P_Y Y = 0.$$

The first two conditions imply that $P_X X = \frac{1}{\lambda}$ and $P_Y Y = \frac{1}{\lambda}$.

The third condition implies that $I - \frac{1}{\lambda} - \frac{1}{\lambda} = 0$, or $\lambda = \frac{2}{I}$.

Substituting this expression into $P_X X = \frac{1}{\lambda}$ and $P_Y Y = \frac{1}{\lambda}$ gives the demand functions:

$$X = \left(\frac{I}{2P_X}\right) \text{ and } Y = \left(\frac{I}{2P_Y}\right).$$

Notice that the demand for each good depends only on the price of that good and on income, not on the price of the other good. Also, the consumer spends exactly half her income on each good, regardless of the prices of the goods.

b. To find the demand functions for X and Y, corresponding to $U(X,Y) = (XY)^{0.5} = (X^{0.5})(Y^{0.5})$, first write out the Lagrangian function:

$$\Phi = (X)^{0.5}(Y)^{0.5} - \lambda(P_X X + P_Y Y - I)$$

Differentiating with respect to X, Y, λ and setting the derivatives equal to zero:

$$\frac{\partial\Phi}{\partial X} = 0.5X^{-0.5}Y^{0.5} - \lambda P_X = 0$$

$$\frac{\partial\Phi}{\partial Y} = 0.5X^{0.5}Y^{-0.5} - \lambda P_Y = 0$$

$$\frac{\partial\Phi}{\partial Y} = I - P_X X - P_Y Y = 0$$

Take the first two conditions, move the terms involving λ to the right hand sides, and then divide the first condition by the second. After some algebra, you'll find $\frac{Y}{X} = \frac{P_X}{P_Y}$, or $P_Y Y = P_X X$. Substitute for $P_Y Y$ in the third condition, which yields $I = 2P_X X$. Therefore, $X = \left(\frac{I}{2P_X}\right)$ and $Y = \left(\frac{I}{2P_Y}\right)$, which are the same demand functions we found for the other utility function.

3. Assume that a utility function is given by Min(X, Y), as in Exercise 1(c). What is the Slutsky equation that decomposes the change in the demand for X in response to a change in its price? What is the income effect? What is the substitution effect?

The full Slutsky equation is $dX/dP_X = \partial X/\partial P_X |_{U=U^*} - X(\partial X/\partial I)$, where the first term on the right represents the substitution effect and the second term represents the income effect. Because there is no substitution effect as price changes with this type of fixed proportions utility function, the substitution effect is zero. Therefore, the Slutsky equation for the fixed proportions utility function is $dX/dP_X = -X(\partial X/\partial I)$. A numerical example will help explain how this works. Suppose the consumer originally purchases 10 units of *X*, and we know that he would buy 1 more unit if his income increased by \$5 (so that $\partial X/\partial I = 1/\$5 = 0.2$). Using the Slutsky equation, $dX/dP_X = -10(0.2) = -2$. Therefore, if the price of *X* increased by \$1, the consumer would buy 2 fewer units of *X*, which would be due solely to the income effect. Conversely, if the price of *X* decreased by \$1, the consumer would buy 2 more units.

Figure 4A.3 below shows that when the price of *X* falls, the consumer's budget line pivots out from L_1 to L_2. A parallel shift of the new budget line back to the original indifference curve, U_1, gives us the hypothetical budget line L_3 from which we determine the substitution effect. Because the consumer would purchase the same bundle of *X* and *Y* as he did along the original budget line, the substitution effect is zero. The income effect is determined by the shift from budget line L_3 to L_2, which results in an increase in utility from U_1 to U_2 and an increase in consumption of *X*.

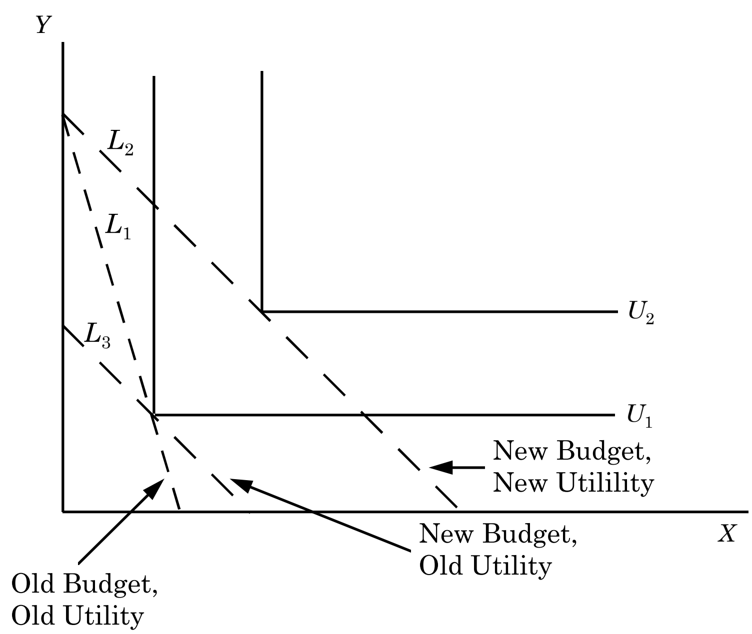


Figure 4A.3

4. Sharon has the following utility function:

$$U(X,Y) = \sqrt{X} + \sqrt{Y}$$

where X is her consumption of candy bars, with price P_X = \$1, and Y is her consumption of espressos, with P_Y = \$3.

a. Derive Sharon's demand for candy bars and espressos.

Using the Lagrangian method, the Lagrangian equation is

$$\Phi = \sqrt{X} + \sqrt{Y} - \lambda(P_X X + P_Y Y - I).$$

To find the demand functions, we need to maximize the Lagrangian equation with respect to X, Y, and λ, which is the same as maximizing utility subject to the budget constraint. The necessary conditions for a maximum are

(1) $$\frac{\partial \Phi}{\partial X} = 0.5X^{-0.5} - P_X\lambda = 0$$

(2) $$\frac{\partial \Phi}{\partial Y} = 0.5Y^{-0.5} - P_Y\lambda = 0$$

(3) $$\frac{\partial \Phi}{\partial \lambda} = I - P_X X - P_Y Y = 0.$$

Combining conditions (1) and (2) results in

$\lambda = \dfrac{1}{2P_X X^{0.5}} = \dfrac{1}{2P_Y Y^{0.5}}$, so that $P_X X^{0.5} = P_Y Y^{0.5}$, and therefore

(4) $$X = \left(\frac{P_Y^2}{P_X^2}\right)Y.$$

Now substitute (4) into (3) and solve for Y. Once you have solved for Y, you can substitute Y back into (4) and solve for X. Note that algebraically there are several ways to solve this type of problem; it does not have to be done exactly as shown here. The demand functions are:

$$Y = \frac{P_X I}{P_Y^2 + P_Y P_X} \text{ or } Y = \frac{I}{12}$$

$$X = \frac{P_Y I}{P_X^2 + P_Y P_X} \text{ or } X = \frac{3I}{4}.$$

b. Assume that her income I = \$100. How many candy bars and how many espressos will Sharon consume?

Substitute the values for the two prices and income into the demand functions to find that she consumes X = 75 candy bars and Y = 8.33 espressos.

c. What is the marginal utility of income?

As shown in the appendix, the marginal utility of income equals 8. From part a, $\lambda = \dfrac{1}{2P_X X^{0.5}} = \dfrac{1}{2P_Y Y^{0.5}}$. Substitute into either part of the equation to get λ = 0.058. This is how much Sharon's utility would increase if she had one more dollar to spend.

5. Maurice has the following utility function: $U(X,Y) = 20X + 80Y - X^2 - 2Y^2$, **where X is his consumption of CDs, with a price of \$1, and Y is his consumption of movie videos, with a rental price of \$2. He plans to spend \$41 on both forms of entertainment. Determine the number of CDs and video rentals that will maximize Maurice's utility.**

Using X as the number of CDs and Y as the number of video rentals, the Lagrangian equation is

$$\Phi = 20X + 80Y - X^2 - 2Y^2 - \lambda(X + 2Y - 41).$$

To find the optimal consumption of each good, maximize the Lagrangian equation with respect to X, Y and λ, which is the same as maximizing utility subject to the budget constraint. The necessary conditions for a maximum are

(1) $$\frac{\partial \Phi}{\partial X} = 20 - 2X - \lambda = 0$$

(2) $$\frac{\partial \Phi}{\partial Y} = 80 - 4Y - 2\lambda = 0$$

(3) $$\frac{\partial \Phi}{\partial \lambda} = X + 2Y - 41 = 0.$$

Note that in condition (3), both sides have been multiplied by –1. Combining conditions (1) and (2) results in

$$\lambda = 20 - 2X = 40 - 2Y$$

(4) $$2Y = 20 + 2X.$$

Now substitute (4) into (3) and solve for X. Once you have solved for X, you can substitute this value back into (4) and solve for Y. Note that algebraically there are several ways to solve this type of problem, and that it does not have to be done exactly as here. The optimal bundle is X = 7 and Y = 17.

CHAPTER 5
UNCERTAINTY AND CONSUMER BEHAVIOR

TEACHING NOTES

This is a useful chapter for business-oriented courses, particularly if you intend to cover the role of risk in capital markets in Chapter 15. It is also good background for Chapter 17 on asymmetric information. On the other hand, you will not able to cover everything in the book, so if there are other topics you wish to cover instead, you may skip this chapter without disrupting the overall flow of the text.

You might start by asking students how uncertainty affects the decisions made by consumers and firms. Consumers may not know their incomes for sure, they are uncertain about the quality of some of the goods they buy (so it is difficult to know the utility they will receive from purchasing those goods); business firms are uncertain about the demands for their products, the future costs of inputs, and the exchange rate for foreign currencies; investors don't know whether their investments will increase or decrease in value; etc. Then ask how people deal with these risks. For example, there are many types of insurance including auto, life and unemployment, companies offer product warranties and refunds, farmers and other firms can hedge against price uncertainty in futures markets, businesses can hedge foreign exchange risks in forward markets, and investors can diversity.

If students have not previously been exposed to probability, expected value, and variance, the basics are covered in Section 5.1. However, this is a fast run-through even for those who have had a basic statistics course, so you may find Exercises 1 through 5 useful as they provide practice calculating expected value and variance. Many students think of risk as arising from the possibility of loss or injury; they do not consider that risk can also be due to uncertain gains. It is easy to construct simple examples where there are only gains to make the point that risk still exists. For example, with alternative 1 you get nothing if a coin comes up heads and $1000 if it comes up tails. With alternative 2, you get $300 for sure. Alternative 1 is clearly more risky. You can mention that variance (or standard deviation), which is often used as a measure of riskiness, takes into account both gains and losses.

Preferences toward risk depend on the decision maker's von Neumann-Morgenstern utility function, which is different from the utility functions in Chapters 3 and 4. Utility in this chapter has some cardinal properties and depends on the monetary payoff to the decision maker, whereas utility in earlier chapters was ordinal and depended on the amount of various goods consumed. To emphasize this difference, utility is denoted by a lower case *u* in this chapter.

There are a number of issues that trip up students in Chapter 5. Most importantly, students often confuse expected utility and the utility of the expected value. Give them a couple of examples to make sure they understand the difference. For instance, if $u(x)=\sqrt{x}$, the expected utility for alternative 1 in the example above is $Eu = 0.5u(0) + 0.5u(1000) = 0.5\sqrt{0} + 0.5\sqrt{1000} = 15.81$. On the other hand, the expected value for alternative 1 is $E(x) = 0.5(0) + 0.5(1000) = 500$, and the utility of the expected value is therefore $u[E(x)] = \sqrt{500} = 22.36$, which is quite different from the expected utility.

Students also have difficulty understanding Figure 5.4 that illustrates the risk premium. They do not understand why the point on the chord (point F) represents expected utility. You will need to explain this carefully. You should also make sure students understand that even risk averse people take risks. Being risk averse does not mean avoiding all risks. Everyone takes risks when the possible rewards are greater than the costs. For instance, most people have parked illegally when they are in a hurry (or late for an exam). We all drive cars, risking accidents and injury, and many people buy stocks and bonds even though those investments may decrease in value. In fact, there really is no way to live and avoid all risks.

Even if your students have not fully understood the technical aspects of choice under uncertainty, they should easily comprehend Examples 5.1 and 5.2 (the latter example leads to Exercise 8).

This is also true of the topics presented in Section 5.3, diversification, insurance and value of information, and Examples 5.3 and 5.4. Also, you might mention the problems of adverse selection and moral hazard in insurance, to be discussed in Chapter 17.

Section 5.4 is more difficult and may be skipped or postponed until after the class has completed the discussion of risk and rates of return in Chapter 15. The last section is a brief introduction to behavioral economics. Most students find this section quite interesting because it considers situations in which people do not behave as rationally as economists typically assume they do.

QUESTIONS FOR REVIEW

1. What does it mean to say that a person is *risk averse*? Why are some people likely to be risk averse while others are risk lovers?

A risk averse person has a diminishing marginal utility of income and prefers a certain income to a gamble with the same expected income. A risk lover has an increasing marginal utility of income and prefers an uncertain income to a certain income when the expected value of the uncertain income equals the certain income. To some extent, a person's risk preferences are like preferences for different vegetables. They may be inborn or learned from parents or others, and we cannot easily say why some people are risk averse while others like taking risks. But there are some economic factors that can affect risk preferences. For example, a wealthy person is more likely to take risks than a moderately well off person, because the wealthy person can better handle losses. Also, people are more likely to take risks when the stakes are low (like office pools around NCAA basketball time) than when stakes are high (like losing a house to fire).

2. Why is the variance a better measure of variability than the range?

Range is the difference between the highest possible outcome and the lowest possible outcome. Range ignores all outcomes except the highest and lowest, and it does not consider how likely each outcome is. Variance, on the other hand, is based on all the outcomes and how likely they are to occur. Variance weights the difference of each outcome from the mean outcome by its probability and, thus, is a more comprehensive measure of variability than the range.

3. George has $5000 to invest in a mutual fund. The expected return on mutual fund A is 15 percent and the expected return on mutual fund B is 10 percent. Should George pick mutual fund A or fund B?

George's decision will depend not only on the expected return for each fund, but also on the variability of each fund's returns and on George's risk preferences. For example, if fund A has a higher standard deviation than fund B, and George is risk averse, then he may prefer fund B even though it has a lower expected return. If George is not particularly risk averse he may choose fund A even if its return is more variable.

4. What does it mean for consumers to maximize expected utility? Can you think of a case in which a person might *not* maximize expected utility?

To maximize expected utility means that the individual chooses the option that yields the highest average utility, where average utility is the probability-weighted sum of all utilities. This theory requires that the consumer knows each possible outcome that may occur and the probability of each outcome. Sometimes consumers either do not know all possible outcomes and the relevant probabilities, or they have difficulty evaluating low-probability, extreme-payoff events.

In some cases, consumers cannot assign a utility level to these extreme-payoff events, such as when the payoff is the loss of the consumer's life. In cases like this, consumers may make choices based on other criteria such as risk avoidance.

5. Why do people often want to insure fully against uncertain situations even when the premium paid exceeds the expected value of the loss being insured against?

Risk averse people have declining marginal utility, and this means that the pain of a loss increases at an increasing rate as the size of the loss increases. As a result, they are willing to pay more than the expected value of the loss to insure against suffering the loss. For example, suppose a homeowner does not insure his house that is worth $200,000. Also suppose there is a small .001 probability that the house will burn to the ground and be a total loss. This means there is a high probability of .999 that there will be no loss. The expected loss is .001(200,000) + .999(0) = $200. Many risk averse homeowners would be willing to pay a lot more than $200 (like $400 or $500) to buy insurance that will replace the house if it burns. They do this because the disutility of losing their $200,000 house is more than 1000 times larger than the disutility of paying the insurance premium.

6. Why is an insurance company likely to behave as if it were risk neutral even if its managers are risk-averse individuals?

A large insurance company sells hundreds of thousands of policies, and the company's managers know they will have to pay for losses incurred by some of their policyholders even though they do not know which particular policies will result in claims. Because of the law of large numbers, however, the company can estimate the total number of claims quite accurately. Therefore, it can make very precise estimates of the total amount it will have to pay in claims. This means the company faces very little risk overall and consequently behaves essentially as if it were risk neutral. Each manager, on the other hand, cannot diversify his or her own personal risks to the same extent, and thus each faces greater risk and behaves in a much more risk averse manner.

7. When is it worth paying to obtain more information to reduce uncertainty?

It is worth paying for information if the information leads the consumer to make different choices than she would have made without the information, and the expected utility of the payoffs (deducting the cost of the information) is greater with the information than the expected utility of the payoffs received when making the best choices without knowing the information.

8. How does the diversification of an investor's portfolio avoid risk?

An investor reduces risk by investing in many assets whose returns are not highly correlated and, even better, some whose returns are negatively correlated. A mutual fund, for example, is a portfolio of stocks of many different companies. If the rate of return on each company's stock is not highly related to the rates of return earned on the other stocks in the portfolio, the portfolio will have a lower variance than any of the individual stocks. This occurs because low returns on some stocks tend to be offset by high returns on others. As the number of stocks in the portfolio increases, the portfolio's variance decreases. While there is less risk in a portfolio of stocks, risk cannot be completely avoided; there is still some market risk in holding a portfolio of stocks compared to a low-risk asset, such as a U.S. government savings bond.

9. Why do some investors put a large portion of their portfolios into risky asset, while others invest largely in risk-free alternatives? (*Hint*: Do the two investors receive exactly the same return on average? If so, why?)

Most investors are risk averse, but some are more risk averse than others. Investors who are highly risk averse will invest largely in risk-free alternatives while those who are less risk averse will put a larger portion of their portfolios into risky assets.

Of course, because investors are risk averse, they will demand higher rates of return on investments that have higher levels of risk (i.e., higher variances). So investors who put larger amounts into risky assets expect to earn greater rates of return than those who invest primarily in risk-free assets.

10. What is an endowment effect? Give an example of such an effect.

An endowment effect exists if an individual places a higher value on an item that is in her possession as compared to the value she places on the same item when it is not in her possession. For example, some people might refuse to pay $5 for a simple coffee mug but would also refuse to sell the same mug for $5 if they already owned it or had just gotten it for free.

11. Jennifer is shopping and sees an attractive shirt. However, the price of $50 is more than she is willing to pay. A few weeks later, she finds the same shirt on sale for $25 and buys it. When a friend offers her $50 for the shirt, she refuses to sell it. Explain Jennifer's behavior.

To help explain Jennifer's behavior, we need to look at the reference point from which she is making the decision. In the first instance, she does not own the shirt so she is not willing to pay the $50 to buy the shirt. In the second instance, she will not accept $50 for the shirt from her friend because her reference point has changed. Once she owns the shirt, the value she attaches to it increases. Individuals often value goods more when they own them than when they do not. This is called the endowment effect.

EXERCISES

1. Consider a lottery with three possible outcomes:

- **$125 will be received with probability .2**
- **$100 will be received with probability .3**
- **$50 will be received with probability .5**

a. What is the expected value of the lottery?

The expected value, *EV*, of the lottery is equal to the sum of the returns weighted by their probabilities:

$$EV = (0.2)(\$125) + (0.3)(\$100) + (0.5)(\$50) = \$80.$$

b. What is the variance of the outcomes?

The variance, σ^2, is the sum of the squared deviations from the mean, $80, weighted by their probabilities:

$$\sigma^2 = (0.2)(125 - 80)^2 + (0.3)(100 - 80)^2 + (0.5)(50 - 80)^2 = \$975.$$

c. What would a risk-neutral person pay to play the lottery?

A risk-neutral person would pay the expected value of the lottery: $80.

2. Suppose you have invested in a new computer company whose profitability depends on two factors: (1) whether the U.S. Congress passes a tariff raising the cost of Japanese computers and (2) whether the U.S. economy grows slowly or quickly. What are the four mutually exclusive states of the world that you should be concerned about?

The four mutually exclusive states may be represented as:

	Congress passes tariff	Congress does not pass tariff
Slow growth rate	State 1: Slow growth with tariff	State 2: Slow growth without tariff
Fast growth rate	State 3: Fast growth with tariff	State 4: Fast growth without tariff

3. Richard is deciding whether to buy a state lottery ticket. Each ticket costs $1, and the probability of winning payoffs is given as follows:

Probability	Return
0.50	**$0.00**
0.25	**$1.00**
0.20	**$2.00**
0.05	**$7.50**

a. What is the expected value of Richard's payoff if he buys a lottery ticket? What is the variance?

The expected value of the lottery is equal to the sum of the returns weighted by their probabilities:

$$EV = (0.5)(\$0) + (0.25)(\$1.00) + (0.2)(\$2.00) + (0.05)(\$7.50) = \$1.025$$

The variance is the sum of the squared deviations from the mean, $1.025, weighted by their probabilities:

$$\sigma^2 = (0.5)(0 - 1.025)^2 + (0.25)(1 - 1.025)^2 + (0.2)(2 - 1.025)^2 + (0.05)(7.5 - 1.025)^2, \text{ or}$$

$$\sigma^2 = 2.812.$$

b. Richard's nickname is "No-Risk Rick" because he is an extremely risk-averse individual. Would he buy the ticket?

An extremely risk-averse individual would probably not buy the ticket. Even though the expected value is higher than the price of the ticket, $1.025 > $1.00, the difference is not enough to compensate Rick for the risk. For example, if his wealth is $10 and he buys a $1.00 ticket, he would have $9.00, $10.00, $11.00, and $16.50, respectively, under the four possible outcomes. If his utility function is $U = W^{0.5}$, where W is his wealth, then his expected utility is:

$$EU = (0.5)(9^{0.5}) + (0.25)(10^{0.5}) + (0.2)(11^{0.5}) + (0.05)(16.5^{0.5}) = 3.157.$$

This is less than 3.162, which is his utility if he does not buy the ticket ($U(10) = 10^{0.5} = 3.162$). Therefore, he would not buy the ticket.

c. Richard has been given 1000 lottery tickets. Discuss how you would determine the smallest amount for which he would be willing to sell all 1000 tickets.

With 1000 tickets, Richard's expected payoff is $1025. He does not pay for the tickets, so he cannot lose money, but there is a wide range of possible payoffs he might receive ranging from $0 (in the extremely unlikely case that all 1000 tickets pay nothing) to $7500 (in the even more unlikely case that all 1000 tickets pay the top prize of $7.50), and everything in between. Given this variability and Richard's high degree of risk aversion, we know that Richard would be willing to sell all the tickets for less (and perhaps considerably less) than the expected payoff of $1025. More precisely, he would sell the tickets for $1025 minus his risk premium. To find his selling price, we would first have to calculate his expected utility for the lottery winnings. This would be like point F in Figure 5.4, except that in Richard's case there are thousands of possible payoffs, not just two as in the figure. Using his expected utility value, we then would find the certain amount that gives him the same level of utility. This is like the $16,000 income at point C in Figure 5.4. That certain amount is the smallest amount for which he would be willing to sell all 1000 lottery tickets.

d. In the long run, given the price of the lottery tickets and the probability/return table, what do you think the state would do about the lottery?

Given the price of the tickets, the sizes of the payoffs and the probabilities, the lottery is a money loser. The state loses $1.025 – 1.00 = $0.025 (two and a half cents) on every ticket it sells. The state must raise the price of a ticket, reduce some of the payoffs, raise the probability of winning nothing, lower the probabilities of the positive payoffs, or some combination of the above.

4. Suppose an investor is concerned about a business choice in which there are three prospects – the probability and returns are given below:

Probability	Return
0.4	$100
0.3	30
0.3	–30

What is the expected value of the uncertain investment? What is the variance?

The expected value of the return on this investment is

$$EV = (0.4)(100) + (0.3)(30) + (0.3)(-30) = \$40.$$

The variance is

$$\sigma^2 = (0.4)(100 - 40)^2 + (0.3)(30 - 40)^2 + (0.3)(-30 - 40)^2 = 2940.$$

5. You are an insurance agent who must write a policy for a new client named Sam. His company, Society for Creative Alternatives to Mayonnaise (SCAM), is working on a low-fat, low-cholesterol mayonnaise substitute for the sandwich-condiment industry. The sandwich industry will pay top dollar to the first inventor to patent such a mayonnaise substitute. Sam's SCAM seems like a very risky proposition to you. You have calculated his possible returns table as follows:

Probability	Return	Outcome
.999	–$1,000,000	(he fails)
.001	$1,000,000,000	(he succeeds and sells his formula)

a. What is the expected return of Sam's project? What is the variance?

The expected return, *ER*, of Sam's investment is

$$ER = (0.999)(-1{,}000{,}000) + (0.001)(1{,}000{,}000{,}000) = \$1000.$$

The variance is

$$\sigma^2 = (0.999)(-1{,}000{,}000 - 1000)^2 + (0.001)(1{,}000{,}000{,}000 - 1000)^2, \text{ or}$$

$$\sigma^2 = 1{,}000{,}998{,}999{,}000{,}000.$$

b. What is the most that Sam is willing to pay for insurance? Assume Sam is risk neutral.

Suppose the insurance guarantees that Sam will receive the expected return of $1000 with certainty regardless of the outcome of his SCAM project. Because Sam is risk neutral and because his expected return is the same as the guaranteed return with insurance, the insurance has no value to Sam. He is just as happy with the uncertain SCAM profits as with the certain outcome guaranteed by the insurance policy. So Sam will not pay anything for the insurance.

c. Suppose you found out that the Japanese are on the verge of introducing their own mayonnaise substitute next month. Sam does not know this and has just turned down your final offer of $1000 for the insurance. Assume that Sam tells you SCAM is only six months away from perfecting its mayonnaise substitute *and* that you know what you know about the Japanese. Would you raise or lower your policy premium on any subsequent proposal to Sam? Based on his information, would Sam accept?

The entry of the Japanese lowers Sam's probability of a high payoff. For example, assume that the probability of the billion-dollar payoff is lowered to zero. Then the expected outcome is:

$$ER = (1.0)(-\$1{,}000{,}000) + (0.0)((\$1{,}000{,}000{,}000) = -\$1{,}000{,}000.$$

Therefore, you should raise the policy premium substantially. But Sam, not knowing about the Japanese entry, will continue to refuse your offers to insure his losses.

6. Suppose that Natasha's utility function is given by $u(I)=\sqrt{10I}$, where *I* represents annual income in thousands of dollars.

a. Is Natasha risk loving, risk neutral, or risk averse? Explain.

Natasha is risk averse. To show this, assume that she has $10,000 and is offered a gamble of a $1000 gain with 50 percent probability and a $1000 loss with 50 percent probability. Her utility of $10,000 is $u(10) = \sqrt{10(10)} = 10$. Her expected utility with the gamble is:

$$EU = (0.5)\sqrt{10(11)} + (0.5)\sqrt{10(9)} = 9.987 < 10.$$

She would avoid the gamble. If she were risk neutral, she would be indifferent between the $10,000 and the gamble, and if she were risk loving, she would prefer the gamble.

You can also see that she is risk averse by noting that the square root function increases at a decreasing rate (the second derivative is negative), implying diminishing marginal utility.

b. Suppose that Natasha is currently earning an income of $40,000 (*I* = 40) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a .6 probability of earning $44,000 and a .4 probability of earning $33,000. Should she take the new job?

The utility of her current salary is $\sqrt{10(40)} = 20$. The expected utility of the new job's salary is

$$EU = (0.6)\sqrt{10(44)} + (0.4)\sqrt{10(33)} = 19.85,$$

which is less than 20. Therefore, she should not take the job. You can also determine that Natasha should reject the job by noting that the expected value of the new job is only $39,600, which is less than her current salary. Since she is risk averse, she should never accept a risky salary with a lower expected value than her current certain salary.

c. In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (*Hint*: What is the risk premium?)

This question assumes that Natasha takes the new job (for some unexplained reason). Her expected salary is 0.6(44,000) + 0.4(33,000) = $39,600. The risk premium is the amount Natasha would be willing to pay so that she receives the expected salary for certain rather than the risky salary in her new job. In part (b) we determined that her new job has an expected utility of 19.85. We need to find the certain salary that gives Natasha the same utility of 19.85, so we want to find I such that $u(I) = 19.85$. Using her utility function, we want to solve the following equation: $\sqrt{10I} = 19.85$. Squaring both sides, $10I$ =394.02, and I = 39.402. So Natasha would be equally happy with a certain salary of $39,402 or the uncertain salary with an expected value of $39,600. Her risk premium is $39,600 – 39,402 = $198. Natasha would be willing to pay $198 to guarantee her income would be $39,600 for certain and eliminate the risk associated with her new job.

7. Suppose that two investments have the same three payoffs, but the probability associated with each payoff differs, as illustrated in the table below:

Payoff	**Probability (Investment A)**	**Probability (Investment B)**
$300	**0.10**	**0.30**
$250	**0.80**	**0.40**
$200	**0.10**	**0.30**

a. Find the expected return and standard deviation of each investment.

The expected value of the return on investment A is

$$EV = (0.1)(300) + (0.8)(250) + (0.1)(200) = \$250.$$

The variance on investment A is

$$\sigma^2 = (0.1)(300 - 250)^2 + (0.8)(250 - 250)^2 + (0.1)(200 - 250)^2 = \$500,$$

and the standard deviation on investment A is $\sigma = \sqrt{500} = \$22.36$.

The expected value of the return on investment B is

$$EV = (0.3)(300) + (0.4)(250) + (0.3)(200) = \$250.$$

The variance on investment B is

$$\sigma^2 = (0.3)(300 - 250)^2 + (0.4)(250 - 250)^2 + (0.3)(200 - 250)^2 = \$1500,$$

and the standard deviation on investment B is $\sigma = \sqrt{1500} = \$38.73$.

b. Jill has the utility function $U = 5I$, where *I* denotes the payoff. Which investment will she choose?

Jill's expected utility from investment A is

$$EU= (0.1)[5(300)] + (0.8)[5(250)] + (0.1)[5(200)] = 1250.$$

Jill's expected utility from investment B is

$$EU=(0.3)[5(300)] + (0.4)[5(250)] + (0.3)[5(200)] = 1250.$$

Since both investments give Jill the same expected utility she will be indifferent between the two. Note that Jill is risk neutral, so she cares only about expected values. Since investments A and B have the same expected values, she is indifferent between them.

c. Ken has the utility function $U = 5\sqrt{I}$. Which investment will he choose?

Ken's expected utility from investment A is

$$EU = (0.1)(5\sqrt{300}) + (0.8)(5\sqrt{250}) + (0.1)(5\sqrt{200}) = 78.98.$$

Ken's expected utility from investment B is

$$EU=(0.3)(5\sqrt{300}) + (0.4)(5\sqrt{250}) + (0.3)(5\sqrt{200}) = 78.82.$$

Ken will choose investment A because it has a slightly higher expected utility. Notice that Ken is risk averse, so he prefers the investment with less variability.

d. Laura has the utility function $U = 5I^2$. Which investment will she choose?

Laura's expected utility from investment A is

$$EU=(0.1)[5(300^2)] + (0.8)[5(250^2)] + (0.1)[5(200^2)] = 315,000.$$

Laura's expected utility from investment B is

$$EU=(0.3)[5(300^2)] + (0.4)[5(250^2)] + (0.3)[5(200^2)] = 320,000.$$

Laura will choose investment B since it has a higher expected utility. Notice that Laura is a risk lover, so she prefers the investment with greater variability.

8. As the owner of a family farm whose wealth is $250,000, you must choose between sitting this season out and investing last year's earnings ($200,000) in a safe money market fund paying 5.0 percent or planting summer corn. Planting costs $200,000, with a six-month time to harvest. If there is rain, planting summer corn will yield $500,000 in revenues at harvest. If there is a drought, planting will yield $50,000 in revenues. As a third choice, you can purchase AgriCorp drought-resistant summer corn at a cost of $250,000 that will yield $500,000 in revenues at harvest if there is rain, and $350,000 in revenues if there is a drought. You are risk averse, and your preference for family wealth (W) is specified by the relationship $U(W)=\sqrt{W}$. The probability of a summer drought is 0.30, while the probability of summer rain is 0.70. Which of the three options should you choose? Explain.

Calculate the expected utility of wealth under the three options. Wealth is equal to the initial $250,000 plus whatever is earned growing corn or investing in the safe financial asset. Expected utility under the safe option, allowing for the fact that your initial wealth is $250,000, is:

$$E(U) = (250{,}000 + 200{,}000(1 + .05))^{.5} = 678.23.$$

Expected utility with regular corn, again including your initial wealth, is:

$E(U) = .7(250{,}000 + (500{,}000 - 200{,}000))^{.5} + .3(250{,}000 + (50{,}000 - 200{,}000))^{.5} = 519.13 + 94.87 = 614.$

Expected utility with drought-resistant corn is:

$E(U) = .7(250{,}000 + (500{,}000 - 250{,}000))^{.5} + .3(250{,}000 + (350{,}000 - 250{,}000))^{.5} = 494.975 + 177.482 = 672.46.$

You should choose the option with the highest expected utility, which is the safe option of not planting corn.

Note: There is a subtle time issue in this problem. The returns from planting corn occur in 6 months while the money market fund pays 5%, which is presumably a yearly interest rate. To put everything on equal footing, we should compare the returns of all three alternatives over a 6-month period. In this case, the money market fund would earn about 2.5%, so its expected utility is:

$$E(U) = (250{,}000 + 200{,}000(1 + .025))^{.5} = 674.54.$$

This is still the best of the three options, but by a smaller margin than before.

9. Draw a utility function over income *u(I)* that describes a man who is a risk lover when his income is low but risk averse when his income is high. Can you explain why such a utility function might reasonably describe a person's preferences?

The utility function will be S-shaped as illustrated below. Preferences might be like this for an individual who needs a certain level of income, I^*, in order to stay alive. An increase in income above I^* will have diminishing marginal utility. Below I^*, the individual will be a risk lover and will take unfavorable gambles in an effort to make large gains in income. Above I^*, the individual will purchase insurance against losses and below I^* will gamble.

10. A city is considering how much to spend to hire people to monitor its parking meters. The following information is available to the city manager:

- **Hiring each meter monitor costs $10,000 per year.**
- **With one monitoring person hired, the probability of a driver getting a ticket each time he or she parks illegally is equal to .25.**
- **With two monitors, the probability of getting a ticket is .5; with three monitors, the probability is .75; and with four, it's equal to 1.**
- **With two monitors hired, the current fine for overtime parking is $20.**

a. Assume first that all drivers are risk neutral. What parking fine would you levy, and how many meter monitors would you hire (1, 2, 3, or 4) to achieve the current level of deterrence against illegal parking at the minimum cost?

If drivers are risk neutral, their behavior is influenced only by their expected fine. With two meter monitors, the probability of detection is 0.5 and the fine is $20. So, the expected fine is (0.5)($20) + (0.5)(0) = $10. To maintain this expected fine, the city can hire one meter monitor and increase the fine to $40, or hire three meter monitors and decrease the fine to $13.33, or hire four meter monitors and decrease the fine to $10.

If the only cost to be minimized is the cost of hiring meter monitors at $10,000 per year you, as the city manager, should minimize the number of meter monitors. Hire only one monitor and increase the fine to $40 to maintain the current level of deterrence.

b. Now assume that drivers are highly risk averse. How would your answer to (a) change?

If drivers are risk averse, they would want to avoid the possibility of paying parking fines even more than would risk neutral drivers. Therefore, a fine of less than $40 with one meter monitor should maintain the current level of deterrence.

c. (For discussion) What if drivers could insure themselves against the risk of parking fines? Would it make good public policy to permit such insurance?

Drivers engage in many forms of behavior to insure themselves against the risk of parking fines, such as checking the time often to be sure they have not parked overtime, parking blocks away from their destination in non-metered spots or taking public transportation. If a private insurance firm offered insurance that paid the fine when a ticket was received, drivers would not worry about getting tickets. They would not seek out unmetered spots or take public transportation; they would park in metered spaces for as long as they wanted at zero personal cost. Having the insurance would lead drivers to get many more parking tickets. This is referred to as moral hazard and may cause the insurance market to collapse, but that's another story (see Section 17.3 in Chapter 17).

It probably would not make good public policy to permit such insurance. Parking is usually metered to encourage efficient use of scarce parking space. People with insurance would have no incentive to use public transportation, seek out-of-the-way parking locations or economize on their use of metered spaces. This imposes a cost on others who are not able to find a place to park. If the parking fines are set to efficiently allocate the scarce amount of parking space available, then the availability of insurance will lead to an inefficient use of the parking space. In this case, it would not be good public policy to permit the insurance.

11. A moderately risk-averse investor has 50 percent of her portfolio invested in stocks and 50 percent in risk-free Treasury bills. Show how each of the following events will affect the investor's budget line and the proportion of stocks in her portfolio:

a. The standard deviation of the return on the stock market increases, but the expected return on the stock market remains the same.

From section 5.4, the equation for the budget line is

$$R_P = \left[\frac{R_m - R_f}{\sigma_m}\right]\sigma_P + R_f,$$

where R_p is the expected return on the portfolio, R_m is the expected return from investing in the stock market, R_f is the risk-free return on Treasury bills, σ_m is the standard deviation of the return from investing in the stock market, and σ_p is the standard deviation of the return on the portfolio. The budget line is linear and shows the positive relationship between the return on the portfolio, R_p, and the standard deviation of the return on the portfolio, σ_p, as shown in Figure 5.6.

In this case σ_m, the standard deviation of the return on the stock market, increases. The slope of the budget line therefore decreases, and the budget line becomes flatter. The budget line's intercept stays the same because R_f does not change. Thus, at any given level of portfolio return, the portfolio now has a higher standard deviation. Since stocks have become riskier without a compensating increase in expected return, the proportion of stocks in the investor's portfolio will fall.

b. The expected return on the stock market increases, but the standard deviation of the stock market remains the same.

In this case, R_m, the expected return on the stock market, increases, so the slope of the budget line becomes steeper. At any given level of portfolio standard deviation, σ_p, there is now a higher expected return, R_p. Stocks have become relatively more attractive because investors now get greater expected returns with no increase in risk, and the proportion of stocks in the investor's portfolio will rise as a consequence.

c. The return on risk-free Treasury bills increases.

In this case there is an increase in R_f, which affects both the intercept and slope of the budget line. The budget line shifts up and become flatter as a result. The proportion of stocks in the portfolio could go either way. On the one hand, Treasury bills now have a higher return and so are more attractive. On the other hand, the investor can now earn a higher return from each Treasury bill and so could hold fewer Treasury bills and still maintain the same level of risk-free return. In this second case, the investor may be willing to place more of her money in the stock market. It will depend on the particular preferences of the investor as well as the magnitude of the returns to the two asset classes. An analogy would be to consider what happens to savings when the interest rate increases. On the one hand, savings tend to increase because the return is higher, but on the other hand, spending may increase and savings decrease because a person can save less each period and still wind up with the same accumulation of savings at some future date.

CHAPTER 6
PRODUCTION

TEACHING NOTES

Chapters 3, 4 and 5 examined consumer behavior and demand. Now, in Chapter 6, we start looking more deeply at supply by studying production. Students often find the theory of supply easier to understand than consumer theory because it is less abstract, and the concepts are more familiar. It is helpful to emphasize the similarities between utility maximization and cost minimization – indifference curves and budget lines become isoquants and isocost lines. Once students have seen consumer theory, production theory usually is a bit easier.

While the concept of a production function is not difficult, the mathematical and graphical representation can sometimes be confusing. Numerical examples are very helpful. Be sure to point out that the production function tells us the *greatest* level of output for any given set of inputs. Thus, engineers have already determined the best production methods for any set of inputs, and all this is captured in the production function. While technical efficiency is assumed throughout, you may want to discuss the importance of improving productivity and the concept of learning by doing, which is covered in Section 7.6 in Chapter 7. Examples 1 and 2 in Chapter 6 are also good for highlighting this issue.

It is important to emphasize that the inputs used in production functions represent *flows* such as labor hours per week. Capital is measured in terms of capital services used during a period of time (e.g., machine hours per month) and not the number of units of capital. Capital flows are especially difficult for students to understand, but it is important to make the point here so that the discussion of input costs in Chapter 7 is easier for students to grasp.

Graphing the one-input production function in Section 6.2 leads naturally to a discussion of marginal product and diminishing marginal returns. Emphasize that diminishing returns exist because some factors are fixed by definition, and that diminishing returns does *not* mean negative returns. If you have not discussed marginal utility, now is the time to make sure that students know the difference between average and marginal. An example that captures students' attention is the relationship between average and marginal test scores. If their latest grade is greater than their average grade to date, it will increase their average.

Isoquants are defined and discussed in Section 6.3 of the chapter. Although the first few sentences in this section suggest that the one-input case corresponds to the short run while the two-input case occurs in the long run, you might want to point out that isoquants can also describe substitution among variable inputs in the short run. For example, skilled and unskilled labor, or labor and raw material can be substituted for each other in the short run. Rely on the students' understanding of indifference curves when discussing isoquants, and point out that, as with indifference curves, isoquants are a two-dimensional representation of a three-dimensional production function. A key concept in this section is the marginal rate of technical substitution, which is like the MRS in consumer theory.

Figure 6.4 is especially useful for demonstrating how diminishing marginal returns depend on the isoquant map. For example, if capital is held constant at 3 units, you can trace out the increase in output as labor increases and see that there are diminishing returns to labor.

Section 6.4 defines returns to scale, which has no counterpart in consumer theory because we do not care about the cardinal properties of utility functions. Be sure to explain the difference between diminishing returns to an input and decreasing returns to scale. Unfortunately, these terms sound very similar and frequently confuse students.

QUESTIONS FOR REVIEW

1. What is a production function? How does a long-run production function differ from a short-run production function?

A production function represents how inputs are transformed into outputs by a firm. In particular, a production function describes the *maximum* output that a firm can produce for each specified combination of inputs. In the short run, one or more factors of production cannot be changed, so a short-run production function tells us the maximum output that can be produced with different amounts of the variable inputs, holding fixed inputs constant. In the long-run production function, all inputs are variable.

2. Why is the marginal product of labor likely to increase initially in the short run as more of the variable input is hired?

The marginal product of labor is likely to increase initially because when there are more workers, each is able to specialize on an aspect of the production process in which he or she is particularly skilled. For example, think of the typical fast food restaurant. If there is only one worker, he will need to prepare the burgers, fries, and sodas, as well as take the orders. Only so many customers can be served in an hour. With two or three workers, each is able to specialize, and the marginal product (number of customers served per hour) is likely to increase as we move from one to two to three workers. Eventually, there will be enough workers and there will be no more gains from specialization. At this point, the marginal product will begin to diminish.

3. Why does production eventually experience diminishing marginal returns to labor in the short run?

The marginal product of labor will eventually diminish because there will be at least one fixed factor of production, such as capital. As more and more labor is used along with a fixed amount of capital, there is less and less capital for each worker to use, and the productivity of additional workers necessarily declines. Think for example of an office where there are only three computers. As more and more employees try to share the computers, the marginal product of each additional employee will diminish.

4. You are an employer seeking to fill a vacant position on an assembly line. Are you more concerned with the average product of labor or the marginal product of labor for the last person hired? If you observe that your average product is just beginning to decline, should you hire any more workers? What does this situation imply about the marginal product of your last worker hired?

In filling a vacant position, you should be concerned with the marginal product of the last worker hired, because the marginal product measures the effect on output, or total product, of hiring another worker. This in turn determines the additional revenue generated by hiring another worker, which should then be compared to the cost of hiring the additional worker.

The point at which the average product begins to decline is the point where average product is equal to marginal product. As more workers are used beyond this point, both average product and marginal product decline. However, marginal product is still positive, so total product continues to increase. Thus, it may still be profitable to hire another worker.

5. What is the difference between a production function and an isoquant?

A production function describes the maximum output that can be achieved with any given combination of inputs. An isoquant identifies all of the different combinations of inputs that can be used to produce one particular level of output.

6. Faced with constantly changing conditions, why would a firm ever keep *any* factors fixed? What criteria determine whether a factor is fixed or variable?

Whether a factor is fixed or variable depends on the time horizon under consideration: all factors are fixed in the very short run while all factors are variable in the long run. As stated in the text, "All fixed inputs in the short run represent outcomes of previous long-run decisions based on estimates of what a firm could profitably produce and sell." Some factors are fixed in the short run, whether the firm likes it or not, simply because it takes time to adjust the levels of those inputs. For example, a lease on a building may legally bind the firm, some employees may have contracts that must be upheld, or construction of a new facility may take a year or more. Recall that the short run is not defined as a specific number of months or years but as that period of time during which some inputs cannot be changed for reasons such as those given above.

7. Isoquants can be convex, linear, or L-shaped. What does each of these shapes tell you about the nature of the production function? What does each of these shapes tell you about the MRTS?

Convex isoquants indicate that some units of one input can be substituted for a unit of the other input while maintaining output at the same level. In this case, the MRTS is diminishing as we move down along the isoquant. This tells us that it becomes more and more difficult to substitute one input for the other while keeping output unchanged. Linear isoquants imply that the slope, or the MRTS, is constant. This means that the same number of units of one input can always be exchanged for a unit of the other input holding output constant. The inputs are perfect substitutes in this case. L-shaped isoquants imply that the inputs are perfect complements, and the firm is producing under a fixed proportions type of technology. In this case the firm cannot give up one input in exchange for the other and still maintain the same level of output. For example, the firm may require exactly 4 units of capital for each unit of labor, in which case one input cannot be substituted for the other.

8. Can an isoquant ever slope upward? Explain.

No. An upward sloping isoquant would mean that if you increased both inputs output would stay the same. This would occur only if one of the inputs reduced output; sort of like a bad in consumer theory. As a general rule, if the firm has more of all inputs it can produce more output.

9. Explain the term "marginal rate of technical substitution." What does a MRTS = 4 mean?

MRTS is the amount by which the quantity of one input can be reduced when the other input is increased by one unit, while maintaining the same level of output. If the MRTS is 4 then one input can be reduced by 4 units as the other is increased by one unit, and output will remain the same.

10. Explain why the marginal rate of technical substitution is likely to diminish as more and more labor is substituted for capital.

As more and more labor is substituted for capital, it becomes increasingly difficult for labor to perform the jobs previously done by capital. Therefore, more units of labor will be required to replace each unit of capital, and the MRTS will diminish. For example, think of employing more and more farm labor while reducing the number of tractor hours used.

At first you would stop using tractors for simpler tasks such as driving around the farm to examine and repair fences or to remove rocks and fallen tree limbs from fields. But eventually, as the number or labor hours increased and the number of tractor hours declined, you would have to plant and harvest your crops primarily by hand. This would take large numbers of additional workers.

11. It is possible to have diminishing returns to a single factor of production and constant returns to scale at the same time. Discuss.

Diminishing returns and returns to scale are completely different concepts, so it is quite possible to have both diminishing returns to, say, labor and constant returns to scale. Diminishing returns to a single factor occurs because all other inputs are fixed. Thus, as more and more of the variable factor is used, the additions to output eventually become smaller and smaller because there are no increases in the other factors. The concept of returns to scale, on the other hand, deals with the increase in output when *all* factors are increased by the same proportion. While each factor by itself exhibits diminishing returns, output may more than double, less than double, or exactly double when *all* the factors are doubled. The distinction again is that with returns to scale, all inputs are increased in the same proportion and no inputs are fixed. The production function in Exercise 10 is an example of a function with diminishing returns to each factor and constant returns to scale.

12. Can a firm have a production function that exhibits increasing returns to scale, constant returns to scale, and decreasing returns to scale as output increases? Discuss.

Many firms have production functions that first exhibit increasing, then constant, and ultimately decreasing returns to scale. At low levels of output, a proportional increase in all inputs may lead to a larger-than-proportional increase in output, because there are many ways to take advantage of greater specialization as the scale of operation increases. As the firm grows, the opportunities for specialization may diminish, and the firm operates at peak efficiency. If the firm wants to double its output, it must duplicate what it is already doing. So it must double all inputs in order to double its output, and thus there are constant returns to scale. At some level of production, the firm will be so large that when inputs are doubled, output will less than double, a situation that can arise from management diseconomies.

13. Give an example of a production process in which the short run involves a day or a week and the long run any period longer than a week.

Suppose a small Mom and Pop business makes specialty teddy bears in the family's garage. It would not take long to hire another worker or buy more supplies; maybe a couple of days. It would take a bit longer to find a larger production facility. The owner(s) would have to look for a larger building to rent or add on to the existing garage. This could easily take more than a week, but perhaps not more than a month or two.

EXERCISES

1. The menu at Joe's coffee shop consists of a variety of coffee drinks, pastries, and sandwiches. The marginal product of an additional worker can be defined as the number of customers that can be served by that worker in a given time period. Joe has been employing one worker, but is considering hiring a second and a third. Explain why the marginal product of the second and third workers might be higher than the first. Why might you expect the marginal product of additional workers to diminish eventually?

The marginal product could well increase for the second and third workers because each would be able to specialize in a different task. If there is only one worker, that person has to take orders and prepare all the food. With 2 or 3, however, one could take orders and the others could do most of the coffee and food preparation.

Eventually, however, as more workers are employed, the marginal product would diminish because there would be a large number of people behind the counter and in the kitchen trying to serve more and more customers with a limited amount of equipment and a fixed building size.

2. Suppose a chair manufacturer is producing in the short run (with its existing plant and equipment). The manufacturer has observed the following levels of production corresponding to different numbers of workers:

Number of chairs	***Number of workers***
1	**10**
2	**18**
3	**24**
4	**28**
5	**30**
6	**28**
7	**25**

a. Calculate the marginal and average product of labor for this production function.

The average product of labor, AP_L, is equal to $\frac{q}{L}$. The marginal product of labor, MP_L, is equal to $\frac{\Delta q}{\Delta L}$, the change in output divided by the change in labor input. For this production process we have:

L	q	AP_L	MP_L
0	0	__	__
1	10	10	10
2	18	9	8
3	24	8	6
4	28	7	4
5	30	6	2
6	28	4.7	–2
7	25	3.6	–3

b. Does this production function exhibit diminishing returns to labor? Explain.

Yes, this production process exhibits diminishing returns to labor. The marginal product of labor, the extra output produced by each additional worker, diminishes as workers are added, and this starts to occur with the second unit of labor.

c. Explain intuitively what might cause the marginal product of labor to become negative.

Labor's negative marginal product for $L > 5$ may arise from congestion in the chair manufacturer's factory. Since more laborers are using the same fixed amount of capital, it is possible that they could get in each other's way, decreasing efficiency and the amount of output. Firms also have to control the quality of their output, and the high congestion of labor may produce products that are not of a high enough quality to be offered for sale, which can contribute to a negative marginal product.

3. Fill in the gaps in the table below.

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	**0**	**–**	**–**
1	**225**		
2			**300**
3		**300**	
4	**1140**		
5		**225**	
6			**225**

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	___	___
1	225	225	225
2	600	375	300
3	900	300	300
4	1140	240	285
5	1365	225	273
6	1350	–15	225

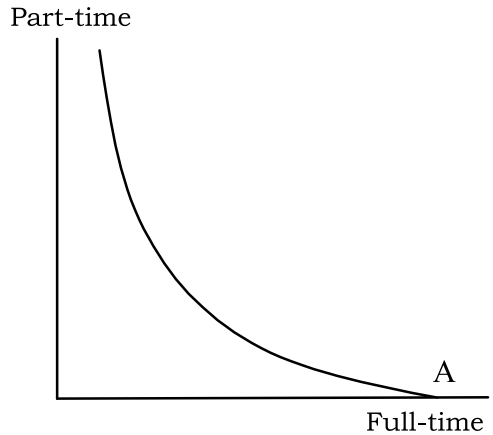
4. A political campaign manager must decide whether to emphasize television advertisements or letters to potential voters in a reelection campaign. Describe the production function for campaign votes. How might information about this function (such as the shape of the isoquants) help the campaign manager to plan strategy?

The output of concern to the campaign manager is the number of votes. The production function has two inputs, television advertising and letters. The use of these inputs requires knowledge of the substitution possibilities between them. If the inputs are perfect substitutes for example, the isoquants are straight lines, and the campaign manager should use only the less expensive input in this case. If the inputs are not perfect substitutes, the isoquants will have a convex shape. The campaign manager should then spend the campaign's budget on the combination of the two inputs will that maximize the number of votes.

5. For each of the following examples, draw a representative isoquant. What can you say about the marginal rate of technical substitution in each case?

a. A firm can hire only full-time employees to produce its output, or it can hire some combination of full-time and part-time employees. For each full-time worker let go, the firm must hire an increasing number of temporary employees to maintain the same level of output.

Place part-time workers on the vertical axis and full-time workers on the horizontal. The slope of the isoquant measures the number of part-time workers that can be exchanged for a full-time worker while still maintaining output. At the bottom end of the isoquant, at point A, the isoquant hits the full-time axis because it is possible to produce with full-time workers only and no part-timers. As we move up the isoquant and give up full-time workers, we must hire more and more part-time workers to replace each full-time worker. The slope increases (in absolute value) as we move up the isoquant. The isoquant is therefore convex and there is a diminishing marginal rate of technical substitution.



b. A firm finds that it can always trade two units of labor for one unit of capital and still keep output constant.

The marginal rate of technical substitution measures the number of units of capital that can be exchanged for a unit of labor while still maintaining output. If the firm can always trade two units of labor for one unit of capital then the MRTS of labor for capital is constant and equal to 1/2, and the isoquant is linear.

c. A firm requires exactly two full-time workers to operate each piece of machinery in the factory

This firm operates under a fixed proportions technology, and the isoquants are L-shaped. The firm cannot substitute any labor for capital and still maintain output because it must maintain a fixed 2:1 ratio of labor to capital. The MRTS is infinite (or undefined) along the vertical part of the isoquant and zero on the horizontal part.

6. A firm has a production process in which the inputs to production are perfectly substitutable in the long run. Can you tell whether the marginal rate of technical substitution is high or low, or is further information necessary? Discuss.

Further information is necessary. The marginal rate of technical substitution, *MRTS,* is the absolute value of the slope of an isoquant. If the inputs are perfect substitutes, the isoquants will be linear. To calculate the slope of the isoquant, and hence the *MRTS*, we need to know the rate at which one input may be substituted for the other. In this case, we do not know whether the MRTS is high or low. All we know is that it is a constant number. We need to know the marginal product of each input to determine the MRTS.

7. The marginal product of labor in the production of computer chips is 50 chips per hour. The marginal rate of technical substitution of hours of labor for hours of machine capital is 1/4. What is the marginal product of capital?

The marginal rate of technical substitution is defined at the ratio of the two marginal products. Here, we are given the marginal product of labor and the marginal rate of technical substitution. To determine the marginal product of capital, substitute the given values for the marginal product of labor and the marginal rate of technical substitution into the following formula:

$$\frac{MP_L}{MP_K} = MRTS, \ or \frac{50}{MP_K} = \frac{1}{4}.$$

Therefore, $MP_K = 200$ computer chips per hour.

8. Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased and the other factor held constant?

a. $q = 3L + 2K$

This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 10. If L is 4 and K is 4 then q is 20. When the inputs are doubled, output will double. Each marginal product is constant for this production function. When L increases by 1, q will increase by 3. When K increases by 1, q will increase by 2.

b. $q = (2L + 2K)^{\frac{1}{2}}$

This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 2.8. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output increases by less than double. The marginal product of each input is decreasing. This can be determined using calculus by differentiating the production function with respect to either input, while holding the other input constant. For example, the marginal product of labor is

$$\frac{\partial q}{\partial L} = \frac{2}{2(2L + 2K)^{\frac{1}{2}}}.$$

Since L is in the denominator, as L gets bigger, the marginal product gets smaller. If you do not know calculus, you can choose several values for L (holding K fixed at some level), find the corresponding q values and see how the marginal product changes. For example, if L=4 and K=4 then q=4. If L=5 and K=4 then q=4.24. If L=6 and K=4 then q= 4.47. Marginal product of labor falls from 0.24 to 0.23. Thus, MP_L decreases as L increases, holding K constant at 4 units.

c. $q = 3LK^2$

This function exhibits increasing returns to scale. For example, if L is 2 and K is 2, then q is 24. If L is 4 and K is 4 then q is 192. When the inputs are doubled, output more than doubles. Notice also that if we increase each input by the same factor λ then we get the following:

$$q' = 3(\lambda L)(\lambda K)^2 = \lambda^3 3LK^2 = \lambda^3 q.$$

Since λ is raised to a power greater than 1, we have increasing returns to scale.

The marginal product of labor is constant and the marginal product of capital is increasing. For any given value of K, when L is increased by 1 unit, q will go up by $3K^2$ units, which is a constant number. Using calculus, the marginal product of capital is $MP_K = 6LK$. As K increases, MP_K increases. If you do not know calculus, you can fix the value of L, choose a starting value for K, and find q. Now increase K by 1 unit and find the new q. Do this a few more times and you can calculate marginal product. This was done in part (b) above, and in part (d) below.

d. $q = L^{\frac{1}{2}}K^{\frac{1}{2}}$

This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 2. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output will exactly double. Notice also that if we increase each input by the same factor, λ, then we get the following:

$$q' = (\lambda L)^{\frac{1}{2}}(\lambda K)^{\frac{1}{2}} = \lambda L^{\frac{1}{2}}K^{\frac{1}{2}} = \lambda q.$$

Since λ is raised to the power 1, there are constant returns to scale.

The marginal product of labor is decreasing and the marginal product of capital is decreasing. Using calculus, the marginal product of capital is

$$MP_K = \frac{L^{\frac{1}{2}}}{2K^{\frac{1}{2}}}.$$

For any given value of L, as K increases, MP_K will decrease. If you do not know calculus then you can fix the value of L, choose a starting value for K, and find q. Let L=4 for example. If K is 4 then q is 4, if K is 5 then q is 4.47, and if K is 6 then q is 4.90. The marginal product of the 5^{th} unit of K is 4.47−4 = 0.47, and the marginal product of the 6^{th} unit of K is 4.90−4.47 = 0.43. Hence we have diminishing marginal product of capital. You can do the same thing for the marginal product of labor.

e. $q = 4L^{\frac{1}{2}} + 4K$

This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 13.66. If L is 4 and K is 4 then q is 24. When the inputs are doubled, output increases by less than double.

The marginal product of labor is decreasing and the marginal product of capital is constant. For any given value of L, when K is increased by 1 unit, q goes up by 4 units, which is a constant number. To see that the marginal product of labor is decreasing, fix K=1 and choose values for L. If L=1 then q=8, if L=2 then q=9.66, and if L=3 then q=10.93. The marginal product of the second unit of labor is 9.66–8=1.66, and the marginal product of the third unit of labor is 10.93–9.66=1.27. Marginal product of labor is diminishing.

9. The production function for the personal computers of DISK, Inc., is given by $q = 10K^{0.5}L^{0.5}$, where q is the number of computers produced per day, K is hours of machine time, and L is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function $q = 10K^{0.6}L^{0.4}$.

► **Note**: The answer at the end of the book (first printing) incorrectly listed this as the answer for Exercise 8. Also, the answer at the end of the book for part (a) is correct only if K = L for both firms. A more complete answer is given below.

a. If both companies use the same amounts of capital and labor, which will generate more output?

Let q_1 be the output of DISK, Inc., q_2, be the output of FLOPPY, Inc., and X be the same equal amounts of capital and labor for the two firms. Then, according to their production functions,

$$q_1 = 10X^{0.5}X^{0.5} = 10X^{(0.5 + 0.5)} = 10X$$

and

$$q_2 = 10X^{0.6}X^{0.4} = 10X^{(0.6 + 0.4)} = 10X.$$

Because $q_1 = q_2$, both firms generate the same output with the same inputs. Note that if the two firms both used the same amount of capital and the same amount of labor, but the amount of capital was not equal to the amount of labor, then the two firms would not produce the same levels of output. In fact, if K > L then $q_2 > q_1$, and if L > K then $q_1 > q_2$.

b. Assume that capital is limited to 9 machine hours, but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.

With capital limited to 9 machine hours, the production functions become $q_1 = 30L^{0.5}$ and $q_2 = 37.37L^{0.4}$. To determine the production function with the highest marginal productivity of labor, consider the following table:

L	q Firm 1	MP_L Firm 1	q Firm 2	MP_L Firm 2
0	0.0	___	0.00	___
1	30.00	30.00	37.37	37.37
2	42.43	12.43	49.31	11.94
3	51.96	9.53	57.99	8.68
4	60.00	8.04	65.06	7.07

For each unit of labor above 1, the marginal productivity of labor is greater for the first firm, DISK, Inc.

If you know calculus, you can determine the exact point at which the marginal products are equal. For firm 1, $MP_L = 15L^{-0.5}$, and for firm 2, $MP_L = 14.95L^{-0.6}$. Setting these marginal products equal to each other,

$$15L^{-0.5} = 14.95L^{-0.6}.$$

Solving for L,

$$L^{0.1} = .997, \text{ or } L = .97.$$

Therefore, for L < .97, MP_L is greater for firm 2 (FLOPPY, Inc.), but for any value of L greater than .97, firm 1 (DISK, Inc.) has the greater marginal productivity of labor.

10. In Example 6.3, wheat is produced according to the production function $q = 100(K^{0.8}L^{0.2})$.

a. Beginning with a capital input of 4 and a labor input of 49, show that the marginal product of labor and the marginal product of capital are both decreasing.

For fixed labor and variable capital:

$K = 4 \Rightarrow q = (100)(4^{0.8})(49^{0.2}) = 660.22$

$K = 5 \Rightarrow q = (100)(5^{0.8})(49^{0.2}) = 789.25 \Rightarrow MP_K = 129.03$

$K = 6 \Rightarrow q = (100)(6^{0.8})(49^{0.2}) = 913.19 \Rightarrow MP_K = 123.94$

$K = 7 \Rightarrow q = (100)(7^{0.8})(49^{0.2}) = 1{,}033.04 \Rightarrow MP_K = 119.85.$

So the marginal product of capital decreases as the amount of capital increases.

For fixed capital and variable labor:

$L = 49 \Rightarrow q = (100)(4^{0.8})(49^{0.2}) = 660.22$

$L = 50 \Rightarrow q = (100)(4^{0.8})(50^{0.2}) = 662.89 \Rightarrow MP_L = 2.67$

$L = 51 \Rightarrow q = (100)(4^{0.8})(51^{0.2}) = 665.52 \Rightarrow MP_L = 2.63$

$L = 52 \Rightarrow q = (100)(4^{0.8})(52^{0.2}) = 668.11 \Rightarrow MP_L = 2.59.$

In this case, the marginal product of labor decreases as the amount of labor increases. Therefore the marginal products of both capital and labor decrease as the variable input increases.

b. Does this production function exhibit increasing, decreasing, or constant returns to scale?

Constant (increasing, decreasing) returns to scale implies that proportionate increases in inputs lead to the same (more than, less than) proportionate increases in output. If we were to increase labor and capital by the same proportionate amount (λ) in this production function, output would change by the same proportionate amount:

$$q' = 100(\lambda K)^{0.8}(\lambda L)^{0.2}, \text{ or}$$

$$q' = 100K^{0.8}L^{0.2}\lambda^{(0.8+0.2)} = \lambda q$$

Therefore, this production function exhibits constant returns to scale. You can also determine this if you plug in values for K and L and compute q, and then double the K and L values to see what happens to q. For example, let K = 4 and L = 10. Then q = 480.45. Now double both inputs to K = 8 and L = 20. The new value for q is 960.90, which is exactly twice as much output. Thus, there are constant returns to scale.

CHAPTER 7
THE COST OF PRODUCTION

TEACHING NOTES

This chapter is packed with new terms and concepts, and it will take some time to go through carefully. You might remind students that you are still building the underpinnings of supply and, for that purpose, it is critical to understand how firms make production decisions. Some key topics in the chapter are accounting versus economic costs; total, average and marginal costs in the short and long run; and choosing the least-cost combination of inputs (graphically in the chapter, and mathematically in the appendix).

Other topics include economies of scope, the learning curve and estimation of cost functions. You may omit some of these additional topics without disrupting the flow of the book if you want to save time for other issues later in the course.

To get started, it is important to distinguish between accounting and economic costs so that students will understand that zero (economic) profit is a reasonable long-run equilibrium in perfect competition (Chapter 8). Opportunity cost is crucial for understanding this distinction. Two examples given in the chapter are the opportunity cost of a business owner's time and the opportunity cost of utilizing capital. The opportunity cost of capital, i.e., the rental rate on capital, may well be the same whether the firm owns or rents the capital and is a source of confusion. It is important, therefore, to distinguish between the purchase price of capital equipment (or its depreciation as determined by accounting rules) and the opportunity cost of using the equipment. Also remind students that the rental rate on capital is the cost for the *flow of capital services* provided by the capital, not the total cost to purchase the capital. Give lots of examples. It is also useful to note that most costs are pretty straightforward explicit costs and are recognized as costs by both accountants and economists.

Sunk costs can also cause problems for students, and many confuse them with fixed costs. The difference is that fixed costs do not vary with output in the short run, but can be reduced or eliminated in the long run. Sunk costs have already been incurred (or committed to) and cannot be recovered or reduced. I like to give examples where people or firms incorrectly take sunk costs into account when making decisions. For instance, investors who will not sell stock that has declined in value until they at least "break even," and companies abandoning projects on which much has already been invested because the **total** amount spent will never be covered by future revenues. Another intriguing example is when a person attends a concert, even though the weather is terrible, because he paid for the ticket, but would not have attended if the ticket had been free. This relates to some of the behavioral economics material in Section 5.5 of Chapter 5.

Following the discussion of opportunity cost, the chapter looks at short-run costs. While the definitions of total, variable, fixed, average and marginal costs and their graphical relationships can seem tedious and/or uninteresting to the student (and some instructors), they are important for understanding the derivation of the firm's supply curve in Chapter 8. Doing algebraic or numerical examples in table form is helpful for most students. Explain that each firm has a unique set of cost curves based on its own particular production function and the prices it has to pay for inputs. Discuss the importance of diminishing returns in explaining the shapes of the short-run cost curves. Point out that average total cost tends to be U-shaped in the short run, that marginal cost intersects average cost and average variable cost at their respective minimum points, and that the minimum of AVC occurs to the left of the minimum of ATC as in Figure 7.1. Draw these curves carefully and encourage your students to do the same.

Section 7.3 on long-run costs goes through the firm's cost-minimization problem. I like to point out that this isn't just a long run problem. Even in the short run, firms have many variable inputs and must choose the least-cost combination of them. So the isocost/isoquant diagram is equally valid in the short run. You should rely heavily on the utility maximization material when covering cost minimization. The major difference is that the budget line (i.e., the isocost) is the objective function rather than the constraint and the indifference curve (i.e., the isoquant) is now the constraint and, of course, we are minimizing rather than maximizing.

I like to express the tangency condition for cost minimization in the form given in expression (7.4); that is, $MP_L/w = MP_K/r$. This has a nice intuitive interpretation and is something firms might actually be able to use to ascertain if they are using the least-cost combination of inputs. For example, suppose a roofing firm is using 3 workers and 2 nailing guns and can roof 100 square feet in an hour. If they had one more worker they could roof 120 square feet, or if they had one more nail gun they could roof 110 square feet. Workers are paid $12 per hour and nail guns cost $3 per hour. From this you can estimate the marginal products and determine that the firm is not at the cost-minimizing point. It should use relatively more nail guns.

After covering the cost-minimizing material, you can point out that cost functions are derived by solving the cost-minimization problem repeatedly for different output amounts. For each solution, calculate total cost as $C = wL + rK$, and plot this against output to get the total cost function. You can do this graphically using the firm's expansion path as in Figure 7.6. I like to illustrate this with an expansion path that is not linear, so that I get a nonlinear cost function. In this way, you can talk about economies and diseconomies of scale and how that affects the shape of the cost function. It can also help to distinguish economies of scale from returns to scale: a subtle distinction that often eludes students.

The relationship between short-run and long-run costs is also a difficulty for some students. Part of the problem is that students do not really understand what lies behind long-run costs. You need to emphasize that it takes time to move from one point to another along the long-run average cost curve. Each point represents the lowest average cost *after* all possible adjustments are made. Many students also have trouble with the fact that most points on the LAC curve do not correspond to minimum points on the corresponding SAC curves. For example, if the firm's chosen output is less than the output where LAC is minimized, the firm uses a plant larger than the one whose SAC is minimized at the chosen output. It purposely underutilizes a bigger plant because the bigger plant is more efficient (i.e., has a lower average cost) at the chosen output level than the smaller plant whose SAC is minimized at that output.

QUESTIONS FOR REVIEW

1. A firm pays its accountant an annual retainer of $10,000. Is this an economic cost?

This is an explicit cost of purchasing the services of the accountant, and it is both an economic and an accounting cost. When the firm pays an annual retainer of $10,000, there is a monetary transaction. The accountant trades his or her time in return for money. An annual retainer is an explicit cost and therefore an economic cost.

2. The owner of a small retail store does her own accounting work. How would you measure the opportunity cost of her work?

The economic, or opportunity, cost of doing accounting work is measured by computing the monetary amount that the owner's time would be worth in its *next best use*. For example, if she could do accounting work for some other company instead of her own, her opportunity cost is the amount she could have earned in that alternative employment. Or if she is a great stand-up comic, her opportunity cost is what she could have earned in that occupation instead of doing her own accounting work.

3. Please explain whether the following statements are true or false.

a. If the owner of a business pays himself no salary, then the accounting cost is zero, but the economic cost is positive.

This is True. Since there is no monetary transaction, there is no accounting, or explicit, cost. However, since the owner of the business could be employed elsewhere, there is an economic cost. The economic cost is positive, and reflectsing the opportunity cost of the owner's time. The economic cost is the value of the owner's time in his next best alternative, or the amount that the owner would earn if he took the next best job.

e.b.A firm that has positive accounting profit does not necessarily have positive economic profit.

True. Accounting profit considers only the explicit, monetary costs. Since there may be some opportunity costs that were not considered fully realized as explicit monetary costs, it is possible that when the opportunity costs are added in, economic profit will become negative. This indicates that the firm's resources are not being put to their best use. Subtracting extra costs could make the profit negative, in economic terms.

f.c. If a firm hires a currently unemployed worker, the opportunity cost of utilizing the worker's services is zero.

False. From the firm's point of view, the wage paid to the worker is an explicit cost whether she was previously unemployed or not. The firm's opportunity cost is equal to the wage, because if it did not hire this worker, it would have had to hire someone else at the same wage. The opportunity cost from the worker's point of view is the value of her time, which is not unlikely to be zero. By taking this job, she cannot work at another job or take care of a child or elderly person at home. If her best alternative is working at another job, she gives up the wage she would have earned. If her best alternative is unpaid, such as taking care of a loved one, she will now have to pay someone else to do that job, and the amount she has to pay is her opportunity cost.

5.4. Suppose that labor is the only variable input to the production process. If the marginal cost of production is diminishing as more units of output are produced, what can you say about the marginal product of labor (the variable input)?

The marginal product of labor must be rising increasing. The marginal cost of production measures the extra cost of producing one more unit of output. If this cost is diminishing, then it must be taking fewer units of labor to produce the extra unit of output. If fewer units of labor are required to produce a unit of output, then the marginal product (extra output produced by an extra unit of labor) must be increasing. Note also, that $MC = w/MP_L$, so that if MC is diminishing then MP_L must be increasing for any given w.

5. Suppose a chair manufacturer finds that the marginal rate of technical substitution of capital for labor in her production process is substantially greater than the ratio of the rental rate on machinery to the wage rate for assembly-line labor. How should she alter her use of capital and labor to minimize the cost of production?

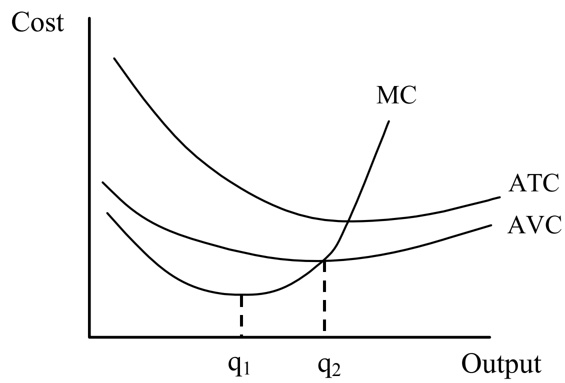
The question states that the MRTS of capital for labor is greater than r/w. Note that this is different from the MRTS of labor for capital, which is what is used in Chapters 6 and 7. The MRTS of labor for capital equals MP_K/MP_L. So, it follows that $MP_K/MP_L > r/w$, or, written another way, $MP_K/r > MP_L/w$. These two ratios should be equal to minimize cost. Since the manufacturer gets more marginal output per dollar from capital than from labor, she should use more capital and less labor to minimize the cost of production.

6. Why are isocost lines straight lines?

The isocost line represents all possible combinations of two inputs that may be purchased for a given total cost. The slope of the isocost line is the negative of the ratio of the input prices. If the input prices are fixed, their ratio is constant and the isocost line is therefore straight. Only if the ratio of the input prices changes as the quantities of the inputs change is the isocost line not straight.

7. Assume that the marginal cost of production is increasing. Can you determine whether the average variable cost is increasing or decreasing? Explain.

When marginal cost is increasing, average variable cost can be either increasing or decreasing as shown in the diagram below. Marginal cost begins increasing at output level q_1, but AVC is decreasing. This happens because MC is below AVC and is therefore pulling AVC down. AVC is decreasing for all output levels between q_1 and q_2. At q_2, MC cuts through the minimum point of AVC, and AVC begins to rise because MC is above it. Thus, for output levels greater than q_2, AVC is increasing.



8. Assume that the marginal cost of production is greater than the average variable cost. Can you determine whether the average variable cost is increasing or decreasing? Explain.

Yes, the average variable cost is increasing. If marginal cost is above average variable cost, each additional unit costs more to produce than the average of the previous units, so the average variable cost is pulled upward. This is shown in the diagram above for output levels greater than q_2.

9. If the firm's average cost curves are U-shaped, why does its average variable cost curve achieve its minimum at a lower level of output than the average total cost curve?

Average total cost is equal to average fixed cost plus average variable cost: ATC = AVC + AFC. When graphed, the difference between the *U*-shaped average total cost and U-shaped average variable cost curves is the average fixed cost, and AFC is downward sloping at all output levels. When AVC is falling, ATC will also fall because both AVC and AFC are declining as output increases. When AVC reaches its minimum (the bottom of its U), ATC will continue to fall because AFC is falling. Even as AVC gradually begins to rise, ATC will still fall because of AFC's decline. Eventually, however, as AVC rises more rapidly, the increases in AVC will outstrip the declines in AFC, and ATC will reach its minimum and then begin to rise.

11.10. If a firm enjoys economies of scale up to a certain output level, and cost then increases proportionately with output, what can you say about the shape of the long-run average cost curve?

When the firm experiences economies of scale, its long-run average cost curve is downward sloping. When costs increase proportionately with output, the firm's long-run average cost curve is horizontal. So this firm's long-run average cost curve has a rounded L-shape; first it falls and then it becomes horizontal as output increases.

11. How does a change in the price of one input change the firm's long-run expansion path?

The expansion path describes the cost-minimizing combination of inputs that the firm chooses for every output level. This combination depends on the ratio of input prices: if the price of one input changes, the price ratio also changes. For example, if the price of an input increases, the intercept of the isocost line on that input's axis moves closer to the origin, and the slope of the isocost line (the price ratio) changes. As the price ratio changes, the firm substitutes away from the now more expensive input toward the cheaper input. Thus, the expansion path bends toward the axis of the now cheaper input.

12. Distinguish between economies of scale and economies of scope. Why can one be present without the other?

Economies of scale refer to the production of *one* good and occur when total cost increases by a smaller proportion than output. Economies of scope refer to the production of *more than one good* and occur when joint production is less costly than the sum of the costs of producing each good or service separately. There is no direct relationship between economies of scale and economies of scope, so production can exhibit one without the other. For example, there are economies of scale producing computers and economies of scale producing carpeting, but if one company produced both, there would probably be no synergies associated with joint production and hence no economies of scope.

16.13. Is the firm's expansion path always a straight line?

No. If the firm always uses capital and labor in the same proportion, the long run expansion path is a straight line. But if the optimal capital-labor ratio changes as output is increased, the expansion path is not a straight line. Also, in the short run the expansion path may be horizontal if capital is fixed.

17.14. What is the difference between economies of scale and returns to scale?

Economies of scale depend on the relationship between what happens to cost and when outpuoutput ! i.e., how does cost change when output is doubled? t is doubled. Returns to scale depend on what happens to output when all inputs are doubled. The difference is that economies of scale reflect input proportions that change optimally as output is increased, while returns to scale are based on fixed input proportions (such as two units of labor for every unit of capital) as output increases.

EXERCISES

1.1. Joe quits his computer programming job, where he was earning a salary of $50,000 per year, to start . He opens his own computer software business store in a building that he owns and was previously renting out for $24,000 per year. In his first year of business he has the following expenses: mortgage $18,000, salary paid to himself, $40,000; rent, $0; other expenses, $25,000. Find the accounting cost and the economic cost associated with Joe's computer software business.

The accounting cost includes only the explicit expenses, which are Joe's salary and his other expenses: 18,000+$40,000 + 25,000 = $8365,000. Economic cost includes these explicit expenses plus opportunity costs. Therefore, economic cost includes the $246,000 he Joe gave up by not renting the building($24,000-$18,000) and an extra $10,000 because he paid himself a salary gave up $10,000 below market on his salary ($50,000 - 40,000). Economic cost is then $40,000 + 25,000 + 24,000 + 10,000 = $99,000.

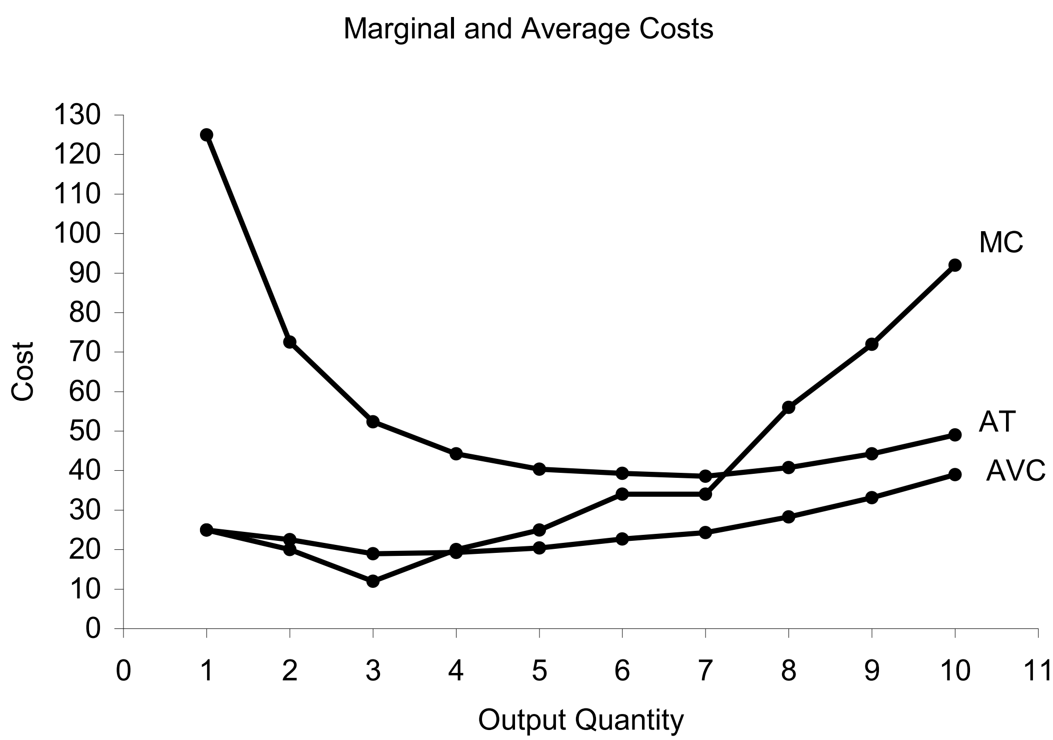
2.2. a. Fill in the blanks in the table on page 262 of the textbook.

Units of Output	Fixed Cost	Variable Cost	Total Cost	Marginal Cost	Average Fixed Cost	Average Variable Cost	Average Total Cost

0	100	0	100	--	--	--	--
1	100	25	125	25	100	25	125
2	100	45	145	20	50	22.50	72.50
3	100	57	157	12	33.33	19.00	52.33
4	100	77	177	20	25.00	19.25	44.25
5	100	102	202	25	20.00	20.40	40.40
6	100	136	236	34	16.67	22.67	39.33
7	100	170	270	34	14.29	24.29	38.57
8	100	226	326	56	12.50	28.25	40.75
9	100	298	398	72	11.11	33.11	44.22
10	100	390	490	92	10.00	39.00	49.00

b. Draw a graph that shows marginal cost, average variable cost, and average total cost, with cost on the vertical axis and quantity on the horizontal axis.

Average total cost is U-shaped and reaches a minimum at an output of about 7. Average variable cost is also U-shaped and reaches a minimum at an output between 3 and 4. Notice that average variable cost is always below average total cost. The difference between the two costs is the average fixed cost. Marginal cost is first diminishing, to a quantity of 3 based on the table, and then increases as q increases. Marginal cost should intersect average variable cost and average total cost at their respective minimum points, though this is not accurately reflected in the table or the graph. If specific functions had been given in the problem instead of just a series of numbers, then it would be possible to find the exact point of intersection between marginal and average total cost and marginal and average variable cost. The curves are likely to intersect at a quantity that is not a whole number, and hence are not listed in the table or represented exactly in the cost diagram.



3. A firm has a fixed production costs of $5,000 and a constant marginal cost of production of equal to $500 per unit produced.

a. What is the firm's total cost function? Average cost?

The variable cost of producing an additional unit, marginal cost, is constant at $500, so $VC = 500q$, and $AVC = \frac{VC}{q} = \frac{500q}{q} = 500$. Fixed cost is $5,000 and therefore average fixed cost is $AFC = \frac{5{,}000}{q}$. The total cost function is fixed cost plus variable cost or $TC = 5{,}000 + 500q$. Average total cost is the sum of average variable cost and average fixed cost: $ATC = 500 + \frac{5{,}000}{q}$.

b. If the firm wanted to minimize the average total cost, would it choose to be very large or very small? Explain.

The firm would choose a very large output because average total cost decreases as q is increased. As q becomes extremely large, ATC will equal approximately 500 because the average fixed cost becomes close to zero.

4. Suppose a firm must pay an annual tax, which is a fixed sum, independent of whether it produces any output.

a. How does this tax affect the firm's fixed, marginal, and average costs?

This tax is a fixed cost because it does not vary with the quantity of output produced. If T is the amount of the tax and F is the firm's original fixed cost, the new total fixed cost increases to $TFC = T + F$. The tax does not affect marginal or variable cost because it does not vary with output. The tax increases both average fixed cost and average total cost by T/q.

b. Now suppose the firm is charged a tax that is proportional to the number of items it produces. Again, how does this tax affect the firm's fixed, marginal, and average costs?

Let t equal the per unit tax. When a tax is imposed on each unit produced, variable cost increases by tq and fixed cost does not change. Average variable cost increases by t, and because fixed costs are constant, average total cost also increases by t. Further, because total cost increases by t for each additional unit produced, marginal cost increases by t.

5. A recent issue of *Business Week* reported the following:

During the recent auto sales slump, GM, Ford, and Chrysler decided it was cheaper to sell cars to rental companies at a loss than to lay off workers. That's because closing and reopening plants is expensive, partly because the auto makers' current union contracts obligate them to pay many workers even if they're not working.

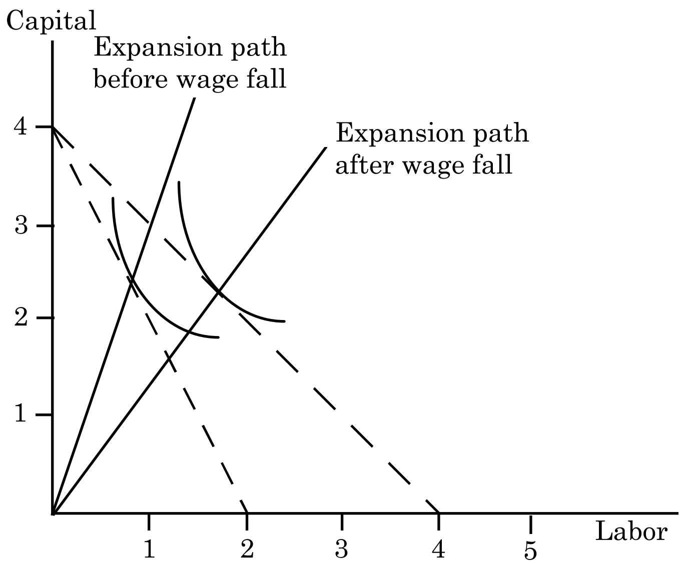
When the article discusses selling cars "at a loss," is it referring to accounting profit or economic profit? How will the two differ in this case? Explain briefly.

When the article refers to the car companies selling at a loss, it is referring to accounting profit. The article is stating that the price obtained for the sale of the cars to the rental companies was less than their accounting cost. Economic profit would be measured by the difference between the price and the opportunity cost of producing the cars. One major difference between accounting and economic cost in this case is the cost of labor. If the car companies must pay many workers even if they are not working, the wages paid to these workers are sunk. If the automakers have no alternative use for these workers (like doing repairs on the factory or preparing the companies' tax returns), the opportunity cost of using them to produce the rental cars is zero. Since the wages would be included in accounting costs, the accounting costs would be higher than the economic costs and would make the accounting profit lower than the economic profit.

6. Suppose the economy takes a downturn, and that labor costs fall by 50 percent and are expected to stay at that level for a long time. Show graphically how this change in the relative price of labor and capital affects the firm's expansion path.

The figure below shows a family of isoquants and two isocost curves. Units of capital are on the vertical axis and units of labor are on the horizontal axis. (*Note*: The figure assumes that the production function underlying the isoquants implies linear expansion paths. However, the results do not depend on this assumption.)

If the price of labor decreases 50% while the price of capital remains constant, the isocost lines pivot outward. Because the expansion path is the set of points where the *MRTS* is equal to the ratio of prices, as the isocost lines become flatter, the expansion path becomes flatter and moves toward the labor axis. As a result the firm uses more labor relative to capital because labor has become less expensive.



8.7. **The cost of flying a passenger plane from point A to point B is $50,000. The airline flies this route four times per day at 7 AM, 10 AM, 1 PM, and 4 PM. The first and last flights are fulfilled l to capacity with 240 people. The second and third flights are only half full. Find the average cost per passenger for each flight. Suppose the airline hires you as a marketing consultant and wants to know which type of customer it should try to attract ! the off-peak customer (the middle two flights) or the rush-hour customer (the first and last flights). What advice would you offer?**

The average cost per passenger is $50,000/240 = $208.33 for the full flights and $50,000/120 = $416.67 for the half full flights. The airline should focus on attracting more off-peak customers because there is excess capacity on the middle two flights. The marginal cost of taking another passenger on those two flights is zero, so the company will increase its profit if it can sell additional tickets for those flights, even if the ticket prices are less than average cost. The peak flights are already full, so attracting more customers at those times will not result in additional ticket sales.

8. You manage a plant that mass-produces engines by teams of workers using assembly machines. The technology is summarized by the production function

$$q = 5\,KL$$

where q is the number of engines per week, K is the number of assembly machines, and L is the number of labor teams. Each assembly machine rents for r = $10,000 per week, and each team costs w = $5000 per week. Engine costs are given by the cost of labor teams and machines, plus $2000 per engine for raw materials. Your plant has a fixed installation of 5 assembly machines as part of its design.

a. What is the cost function for your plant — namely, how much would it cost to produce q engines? What are average and marginal costs for producing q engines? How do average costs vary with output?

The short-run production function is $q = 5(5)L = 25L$, because K is fixed at 5. This implies that for any level of output q, the number of labor teams hired will be

$L = \frac{q}{25}$. The total cost function is thus given by the sum of the costs of capital, labor, and raw materials:

$$TC(q) = rK + wL + 2000q = (10{,}000)(5) + (5{,}000)\left(\frac{q}{25}\right) + 2{,}000q$$

$$TC(q) = 50{,}000 + 2200q.$$

The average cost function is then given by:

$$AC(q) = \frac{TC(q)}{q} = \frac{50{,}000 + 2200q}{q}.$$

and the marginal cost function is given by:

$$MC(q) = \frac{dTC}{dq} = 2200.$$

Marginal costs are constant at $2200 per engine and average costs will decrease as quantity increases because the average fixed cost of capital decreases.

b. How many teams are required to produce 250 engines? What is the average cost per engine?

To produce q = 250 engines we need $L = \frac{q}{25}$ or $L = 10$ labor teams. Average costs are given by

$$AC(q = 250) = \frac{50{,}000 + 2200(250)}{250} = 2400.$$

c. You are asked to make recommendations for the design of a new production facility. What capital/labor *(K/L)* ratio should the new plant accommodate if it wants to minimize the total cost of producing at any level of output q?

We no longer assume that K is fixed at 5. We need to find the combination of K and L that minimizes costs at any level of output q. The cost-minimization rule is given by

$$\frac{MP_K}{r} = \frac{MP_L}{w}.$$

To find the marginal product of capital, observe that increasing K by 1 unit increases q by 5L, so $MP_K = 5L$. Similarly, observe that increasing L by 1 unit increases q by 5K, so $MP_L = 5K$. Mathematically,

$$MP_K = \frac{\partial q}{\partial K} = 5L \text{ and } MP_L = \frac{\partial q}{\partial L} = 5K.$$

Using these formulas in the cost-minimization rule, we obtain:

$$\frac{5L}{r} = \frac{5K}{w} \Rightarrow \frac{K}{L} = \frac{w}{r} = \frac{5000}{10{,}000} = \frac{1}{2}.$$

The new plant should accommodate a capital to labor ratio of 1 to 2, and this is the same regardless of the number of units produced.

9. The short-run cost function of a company is given by the equation TC = 200 + 55*q*, where TC is the total cost and *q* is the total quantity of output, both measured in thousands.

a. What is the company's fixed cost?

When $q = 0$, $TC = 200$, so fixed cost is equal to 200 (or $200,000).

b. If the company produced 100,000 units of goods, what would be its average variable cost?

With 100,000 units, $q = 100$. Variable cost is $55q = (55)(100) = 5500$ (or $5,500,000).

Average variable cost is $\frac{TVC}{q} = \frac{\$5500}{100} = \$55$, or $55,000.

c. What would be its marginal cost of production?

With constant average variable cost, marginal cost is equal to average variable cost, $55 (or $55,000).

d. What would be its average fixed cost?

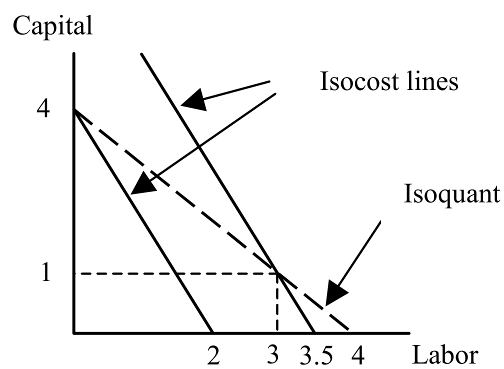
At $q = 100$, average fixed cost is $\frac{TFC}{q} = \frac{\$200}{100} = \$2$ or ($2,000).

e. Suppose the company borrows money and expands its factory. Its fixed cost rises by $50,000, but its variable cost falls to $45,000 per 1000 units. The cost of interest (*i*) also enters into the equation. Each 1-point increase in the interest rate raises costs by $3,000. Write the new cost equation.

Fixed cost changes from 200 to 250, measured in thousands. Variable cost decreases from 55 to 45, also measured in thousands. Fixed cost also includes interest charges: $3i$. The cost equation is

$$TC = 250 + 45q + 3i.$$

10. A chair manufacturer hires its assembly-line labor for $30 an hour and calculates that the rental cost of its machinery is $15 per hour. Suppose that a chair can be produced using 4 hours of labor or machinery in any combination. If the firm is currently using 3 hours of labor for each hour of machine time, is it minimizing its costs of production? If so, why? If not, how can it improve the situation? Graphically illustrate the isoquant and the two isocost lines for the current combination of labor and capital and for the optimal combination of labor and capital.



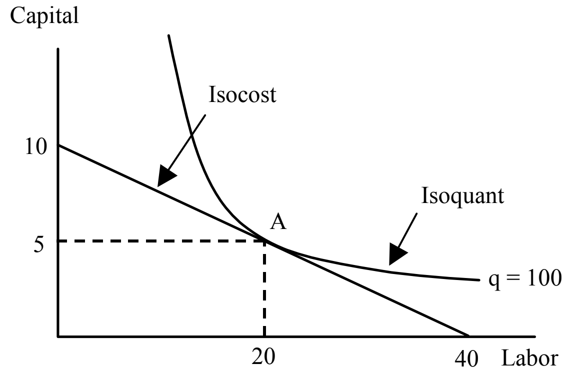
If the firm can produce one chair with either four hours of labor or four hours of machinery (i.e., capital), or any combination, then the isoquant is a straight line with a slope of -1 and intercepts at $K = 4$ and $L = 4$, as depicted by the dashed line.

The isocost lines, $TC = 30L + 15K$, have slopes of !30/15 = !2 when plotted with capital on the vertical axis and intercepts at $K = TC/15$ and $L = TC/30$. The cost minimizing point is the corner solution where $L = 0$ and $K = 4$, so the firm is not currently minimizing its costs. At the optimal point, total cost is $60. Two isocost lines are illustrated on the graph. The first one is further from the origin and represents the current higher cost ($105) of using 3 labor and 1 capital. The firm will find it optimal to move to the second isocost line which is closer to the origin, and which represents a lower cost ($60). In general, the firm wants to be on the lowest isocost line possible, which is the lowest isocost line that still intersects the given isoquant.

11. Suppose that a firm's production function is $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. The cost of a unit of labor is $20 and the cost of a unit of capital is $80.

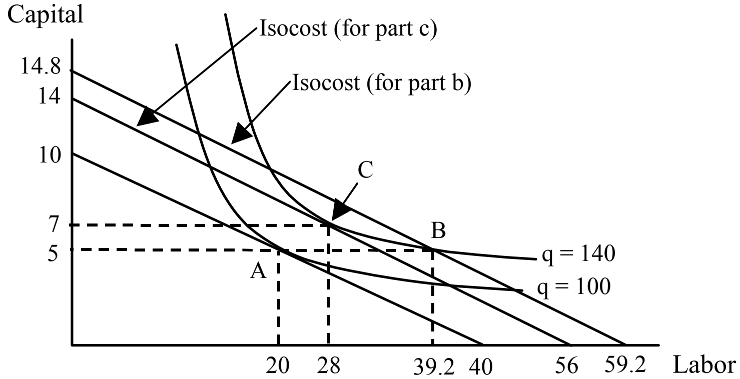
a. The firm is currently producing 100 units of output and has determined that the cost-minimizing optimal quantities of labor and capital are 20 and 5, respectively. Graphically illustrate this using isoquants and isocost lines.

To graph the isoquant, set q = 100 in the production function and solve it for K. After some work, K = 100/L. Choose various combination of L and K and plot them. The isoquant is convex. The optimal quantities of labor and capital are given by the point where the isocost line is tangent to the isoquant. The isocost line has a slope of !1/4, given labor is on the horizontal axis. The total cost is TC = ($20)(20) + ($80)(5) = $800, so the isocost line has the equation 20L + 80K = 800, or K = 10 ! .25L, with intercepts K = 10 and L = 40. The optimal point is labeled A on the graph.



b. The firm now wants to increase output to 140 units. If capital is fixed in the short run, how much labor will the firm requireneed? Illustrate this point graphically and find the firm's new total cost.

The new level of labor is 39.2. To find this, use the production function $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$ and substitute 140 in for output and 5 in for capital; then solve for L. The new cost is TC = ($20)(39.2) + ($80)(5) = $1184. The new isoquant for an output of 140 is above and to the right of the original isoquant. Since capital is fixed in the short run, the firm will move out horizontally to the new isoquant and new level of labor. This is point B on the graph below. This is not the long-run cost minimizing point, but it is the best the firm can do in the short run with K fixed at 5. You can tell that this is not the long-run optimum because the isocost is not tangent to the isoquant at point B. Also there are points on the new (q = 140) isoquant that are below the new isocost (for part b) line. These points all involve hiring more capital and less labor.



c. Graphically identify the optimal cost-minimizing level of capital and labor in the long run if the firm wants to produce 140 units.

This is point C on the graph above. When the firm is at point B it is not minimizing cost. The firm will find it optimal to hire more capital and less labor and move to the new lower isocost (for part c) line that is tangent to the q = 140 isoquant. Note that all three isocost lines are parallel and have the same slope.

d. If the marginal rate of technical substitution is $\frac{K}{L}$, find the optimal level of capital and labor required to produce the 140 units of output.

Set the marginal rate of technical substitution equal to the ratio of the input costs so that $\frac{K}{L} = \frac{20}{80} \Rightarrow K = \frac{L}{4}$. Now substitute this into the production function for K, set q equal to 140, and solve for L: $140 = 10L^{\frac{1}{2}}\left(\frac{L}{4}\right)^{\frac{1}{2}} \Rightarrow L = 28, K = 7$. This is point C on the graph. The new cost is TC = (\$20)(28) + (\$80)(7) = \$1120, which is less than in the short-run (part b), because the firm can adjust all its inputs now.

12. A computer company's cost function, which relates its average cost of production AC to its cumulative output in thousands of computers Q and its plant size in terms of thousands of computers produced per year q (within the production range of 10,000 to 50,000 computers), is given by

AC = 10 ! 0.1Q + 0.3q.

a. Is there a learning curve effect?

The learning curve describes the relationship between the cumulative output and the inputs required to produce a unit of output. Average cost measures the input requirements per unit of output. Learning curve effects exist if average cost falls with increases in cumulative output. Here, average cost decreases as cumulative output, Q, increases. Therefore, there are learning curve effects.

b. Are there economies or diseconomies of scale?

There are diseconomies of scale. Holding cumulative output, Q, constant, there are diseconomies of scale if the firm doubles its rate of output, q, and total cost more than doubles as a result. If this happens, average cost increases as q increases. In this example, average cost increases by \$0.30 for each additional unit produced, so there are diseconomies of scale.

c. During its existence, the firm has produced a total of 40,000 computers and is producing 10,000 computers this year. Next year it plans to increase its production

to 12,000 computers. Will its average cost of production increase or decrease? Explain.

First, calculate average cost this year:

$$AC_1 = 10 - 0.1Q + 0.3q = 10 - (0.1)(40) + (0.3)(10) = 9.$$

Second, calculate the average cost next year:

$$AC_2 = 10 - (0.1)(50) + (0.3)(12) = 8.6.$$

(Note: Cumulative output has increased from 40,000 to 50,000.) The average cost will decrease because of the learning effect, and despite the diseconomies of scale involved when annual output increases from 10 to 12 thousand computers.

13. Suppose the long-run total cost function for an industry is given by the cubic equation TC = a + bq + cq² + dq³. Show (using calculus) that this total cost function is consistent with a U-shaped average cost curve for at least some values of a, b, c, and d.

To show that the cubic cost equation implies a *U*-shaped average cost curve, we use algebra, calculus, and economic reasoning to place sign restrictions on the parameters of the equation. These techniques are illustrated by the example below.

First, if output is equal to zero, then $TC = a$, where a represents fixed costs. In the short run, fixed costs are positive, $a > 0$, but in the long run, where all inputs are variable $a = 0$. Therefore, we restrict a to be zero.

Next, we know that average cost must be positive. Dividing TC by q, with $a = 0$:

$$AC = b + cq + dq^2.$$

This equation is simply a quadratic function. When graphed, it has two basic shapes: a *U* shape and a hill (upside down *U*) shape. We want the *U*, i.e., a curve with a minimum (minimum average cost), rather than a hill with a maximum.

At the minimum, the slope should be zero, thus the first derivative of the average cost curve with respect to q must be equal to zero. For a *U*-shaped AC curve, the second derivative of the average cost curve must be positive.

The first derivative is $c + 2dq$; the second derivative is $2d$. If the second derivative is to be positive, then $d > 0$. If the first derivative is to equal zero, then solving for c as a function of q and d yields: $c = -2dq$. Since d is positive, and the minimum AC must be at some point where q is positive, then c must be negative: $c < 0$.

To restrict b, we know that at its minimum, average cost must be positive. The minimum occurs when $c + 2dq = 0$. We solve for q as a function of c and d: $q = !c/2d > 0$. Next, substituting this value for q into the expression for average cost, and simplifying the equation:

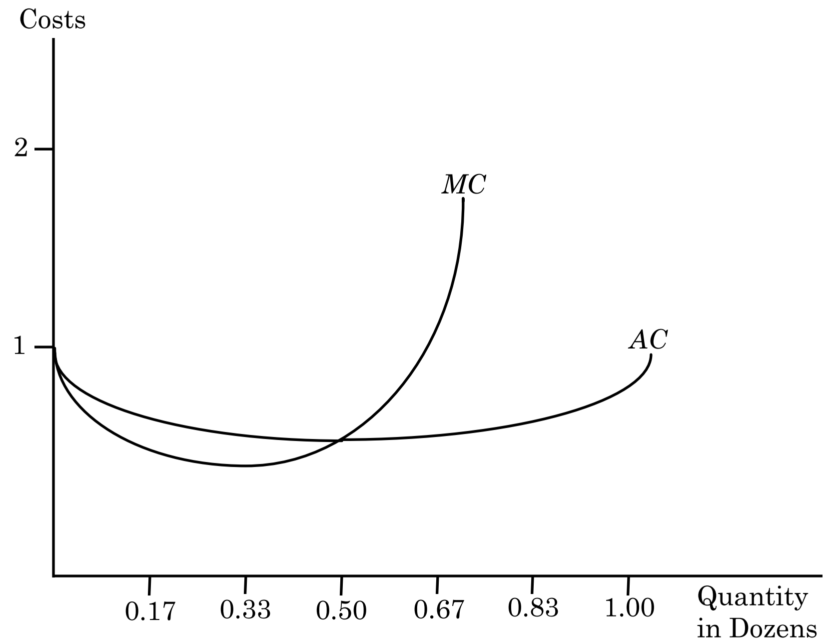
$$AC = b + cq + dq^2 = b + c\left(\frac{-c}{2d}\right) + d\left(\frac{-c}{2d}\right)^2, \text{ or}$$

$$AC = b - \frac{c^2}{2d} + \frac{c^2}{4d} = b - \frac{2c^2}{4d} + \frac{c^2}{4d} = b - \frac{c^2}{4d} > 0.$$

implying $b > \frac{c^2}{4d}$. Because $c^2 > 0$ and $d > 0$, b must be positive.

In summary, for *U*-shaped long-run average cost curves, a must be zero, b and d must be positive, c must be negative, and $4db > c^2$. However, the conditions do not insure that marginal cost is positive. To insure that marginal cost has a *U* shape and that its minimum is positive, use the same procedure, i.e., solve for q at minimum marginal cost: $q = -c/3d$. Then substitute into the expression for marginal cost: $b + 2cq + 3dq^2$. From this we find that c^2 must be less than $3bd$. Notice that parameter values that satisfy this condition also satisfy $4db > c^2$, but not the reverse, so $c^2 < 3bd$ is the more stringent requirement.

For example, let $a = 0$, $b = 1$, $c = -1$, $d = 1$. These values satisfy all the restrictions derived above. Total cost is $q - q^2 + q^3$; average cost is $1 - q + q^2$; and marginal cost is $1 - 2q + 3q^2$. Minimum average cost is where $q = 1/2$ and minimum marginal cost is where $q = 1/3$ (think of q as dozens of units, so no fractional units are produced). See the figure below.



14. A computer company produces hardware and software using the same plant and labor. The total cost of producing computer processing units H and software programs S is given by

$$TC = aH + bS - cHS$$

where *a*, *b*, and *c* are positive. Is this total cost function consistent with the presence of economies or diseconomies of scale? With economies or diseconomies of scope?

If each product were produced by itself there would be neither economies nor diseconomies of scale. To see this, define the total cost of producing H alone (TC_H) to be the total cost when $S = 0$. Thus $TC_H = aH$. Similarly, $TC_S = bS$. In both cases, doubling the number of units produced doubles the total cost, so there are no economies or diseconomies of scale.

Economies of scope exist if $S_C > 0$, where, from equation (7.7) in the text:

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)}.$$

In our case, $C(q_1)$ is TC_H, $C(q_2)$ is TC_S, and $C(q_1,q_2)$ is TC. Therefore,

$$SC = \frac{aH + bS - (aH + bS - cHS)}{aH + bS - cHS} = \frac{cHS}{aH + bS - cHS}.$$

Because cHS (the numerator) and TC (the denominator) are both positive, it follows that $SC > 0$, and there are economies of scope.