习 题 答 案

第 一 章

1-1 $\varphi=\dfrac{ap^2}{l(\omega^2-p^2)}\left(\sin\ pt-\dfrac{p}{\omega}\sin\ \omega t\right)$，其中 $\omega=\sqrt{\dfrac{g}{l}}$

1-2 $a=\dfrac{m_1\sin 2\theta}{2(m_2+m_1\sin^2\theta)}g$

1-3 $R\leqslant\dfrac{fg}{\omega^2}$

1-4 $\tan\theta=\dfrac{g}{\omega^2 r}$

1-5 (1) $y=\dfrac{\omega^2x^2}{2g}$；(2) $h'=h-\dfrac{\omega^2R^2}{4g}$

1-6 $\xi=a\cosh(\omega t)$

$F_N=2m\omega^2 a\sinh(\omega t)$

1-7 $T=2\pi\sqrt{\dfrac{m}{(k-m\omega^2)}}$

1-8 $v_{r0}=l\omega$

1-9 $x_r=\dfrac{1}{2\omega}[(a\omega+v_{r1})e^{\omega t}+(a\omega-v_r)e^{-\omega t}]$

$F_N=m\omega[(a\omega+v_{r1})e^{\omega t}-(a\omega-v_{r1})e^{-\omega t}+\omega\sqrt{R_1^2-a^2}]$

$t_2=\dfrac{1}{\omega}\ln\dfrac{\omega(a+l)+v_{r2}}{a\omega+v_{r1}}$，$v_{r2}=\sqrt{\omega^2(R_2^2-R_1^2)+v_{r1}^2}$

1-10 $\omega_n^2=\dfrac{a\omega^2}{l}$

1-11 $h=1.85$ cm

1-12 (1) $x^2=\dfrac{J(\omega_0-\omega)+m\omega_0 l^2\sin^2\theta}{m\omega\sin^2\theta}$；(2) $\ddot{x}-x\omega^2\sin^2\theta+g\cos\theta=0$

第 二 章

2-1 $F_{av}=799.5$ kN

2-2 $\omega_1=\dfrac{J_O\omega}{J_O+mr^2}$；$v=r\omega_1$；$I=m\dfrac{J_O r}{J_O+mr^2}\cdot\omega$

2-3 $v_1=3.175$ m/s，$\theta=\arctan\dfrac{v_{1n}}{v_{1t}}=19.1°$

$v_2 = 4.157$ m/s，沿撞击点法线方向

2－4 $\omega = \dfrac{mlr\omega_0 \cos\theta}{mr^2 + 3J_{O1}\cos^2\theta}$，$I = \dfrac{J_{O1}\, ml\omega_0 \cos\theta}{mr^2 + 3J_{O1}\cos^2\theta}$；当 $\theta = 90°$ 时，$I = 0$

2－5 $s = \dfrac{3l}{2f}\dfrac{m_1^2}{(m_1 + 3m_2)^2}$

2－6 $\omega = \dfrac{3v}{4a}$

2－7 $k = \sqrt{2}\sin\dfrac{\varphi}{2}$；$x = \dfrac{2}{3}l$

2－8 $\omega = \dfrac{12}{7l}\sqrt{2gh}$；$I = \dfrac{4m}{7}\sqrt{2gh}$

2－9 $v'_C = \dfrac{1 + 2\cos\theta}{3}v_C$；$\omega = \dfrac{1 + 2\cos\theta}{3r}v_C$

$I_n = mv_C \sin\theta$，$I_t = mv_C\dfrac{1 - \cos\theta}{3}$，其中 $\cos\theta = \dfrac{r - h}{r}$

2－10 $\omega = \dfrac{1}{4}\omega_0$

2－11 $h = \dfrac{7}{5}r$

2－12 $\tan\beta = \dfrac{1}{5k}\left(3\tan\theta - \dfrac{2r\omega_0}{v\cos\theta}\right)$

2－13 $\sin\dfrac{\varphi}{2} = \dfrac{\sqrt{3}I}{2m\sqrt{10\,gl}}$

2－14 $v_A = \dfrac{2}{9}\dfrac{I}{m_2}$，方向向左

2－15 $W_1 = 33\,300$ J，$W_2 = 4\,200$ J；$\eta = 89\%$

2－16 $\omega_{BC} = 2.50$ rad/s，顺时针方向

第 三 章

3－1 $\cos\theta = \dfrac{m_2}{4am_1\omega^2 g}$

3－2 (1) $\ddot{\varphi} + \dfrac{g}{l}\sin\varphi = 0$

(2) $l^2[(l^2 - x^2)\ddot{x} + x\dot{x}^2] + gx(l^2 - x^2)^{\frac{3}{2}} = 0$

(3) $l^2[(l^2 - y^2)\ddot{y} + y\dot{y}^2] - g(l^2 - y^2)^2 = 0$

3－3 $(l + R\theta)\ddot{\theta} + R\dot{\theta}^2 + g\sin\theta = 0$

3－4 $\alpha = \dfrac{M}{22mr^2}$

3－5 $\left(\dfrac{m_A}{\tan^2\theta} + m\right)\ddot{s} + ks = \dfrac{F}{\tan\theta} - mg$

3－6 $(1 - \cos\theta)\ddot{\theta} + \dfrac{1}{2}\sin\theta\,\dot{\theta}^2 - \dfrac{g}{2R}\sin\theta = 0$

3-7 $\begin{cases} \ddot{x} - \frac{1}{2}\ddot{\varphi} l\cos(\theta-\varphi) - \frac{1}{2}\dot{\varphi}^2 l\sin(\theta-\varphi) = g\sin\theta \\ -\frac{1}{2}\ddot{x}\cos(\theta-\varphi) + \frac{1}{3}l\ddot{\varphi} + \frac{1}{2}g\sin\varphi = 0 \end{cases}$

当 $t=0$ 时，$F_N = mg\dfrac{\cos\theta}{1+3\sin^2\theta}$

3-8 运动微分方程为：

$$\frac{d}{dt}[J_O\dot{\theta} + m(l+x)^2\dot{\theta}] = 0,\ m\ddot{x} - m(l+x)\dot{\theta}^2 + kx = 0$$

方程的初积分为：

$$\dot{\theta} = \frac{C_1}{J_O + m(l+x)^2}$$

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 - \frac{1}{2}[J_O + m(l+x)^2]\dot{\theta}^2 = C_2$$

其中 C_1 与 C_2 为积分常数

3-9 $\ddot{\theta} - \dfrac{\dot{\varphi}^2}{2}\sin 2\theta + \dfrac{g}{r}\sin\theta = 0$

$$M = mr^2\dot{\theta}\,\dot{\varphi}\sin 2\theta$$

3-10 必须 $m_1 > \dfrac{4m_2m_3}{m_2+m_3}$，重物方能下降，此时

$$F_T = \frac{8m_1m_2m_3}{m_1(m_2+m_3)+4m_2m_3}g$$

3-11 $\dfrac{1}{2}m_2r^2\ddot{\varphi} = M$

$$\left(\frac{1}{3}m_1 + m_2\right)l^2\ddot{\theta} + k\theta - \left(\frac{m_1}{2} + m_2\right)gl\sin\theta = 0$$

3-12 $(m_1+m_2)\ddot{x} + m_2l\ddot{\varphi} + kx = 0,\ \ddot{x} + l\ddot{\varphi} + g\varphi = 0$

3-13 $\rho^2 > r^2\dfrac{m_B}{m_A - m_B}$

3-14 $\alpha_1 = 3.72\ \text{rad/s}^2,\ \alpha_0 = 3.04\ \text{rad/s}^2$

3-15 $(m_1+m_2)\ddot{x} = F(\cos\theta + f\sin\theta) - f(m_1+m_2)g,\ R\ddot{\psi} = 2fg$

$$m_2R\ddot{\varphi} = 2f(m_2g - F\sin\theta)$$

由又滚又滑条件：$\begin{cases} \ddot{x} > R\ddot{\psi} \\ \ddot{x} > R\ddot{\varphi} \end{cases}$ 且右轮不可离开地面，得：$\dfrac{m_2g}{\sin\theta} > F > \dfrac{3f(m_1+m_2)g}{\cos\theta + f\sin\theta}$

3-16 $x_A = -\dfrac{\sqrt{3}}{48}gt^2,\ y_D = \dfrac{3}{16}gt^2,\ \varphi_B = \dfrac{5}{16}\dfrac{g}{r}t^2$

第 四 章

4-1 (a),(b)串联 $T = 2\pi\sqrt{\dfrac{m(k_1+k_2)}{k_1k_2}} = 0.290\ \text{s}$

(c),(d)并联 $T=2\pi\sqrt{\dfrac{m}{k_1+k_2}}=0.140\ \mathrm{s}$

4-2 $k=\dfrac{4\pi^2(m_1-m_2)}{T_1^2-T_2^2}$

4-3 $F=46.68\ \mathrm{kN}$

4-4 $y=(-5\cos 44.3t+100\sin 44.3t)\ \mathrm{mm}$

4-5 $T=2\pi\sqrt{\dfrac{m}{k}}, A=\sqrt{\dfrac{mg}{k}\left(\dfrac{mg\sin^2\theta}{k}+2h\right)}$

4-6 $T=2\pi\sqrt{\dfrac{ml}{2F}}$

4-7 (1) $T=2\pi\sqrt{\dfrac{a}{fg}}$;(2) $f=0.25$

4-8 $\omega_0=\sqrt{\dfrac{2k}{m_1+4m_2}}$

4-9 $\omega_0=\sqrt{\dfrac{6k}{m}-\dfrac{3g}{2l}},\ k>\dfrac{mg}{4l}$

4-10 $\omega_1^2=\dfrac{6k}{7m},\omega_2^2=\dfrac{6}{7}\left(\dfrac{k}{m}+\dfrac{2g}{l}\right)$

4-11 (1) $\omega_0=\sqrt{\dfrac{3k}{2ml^2}}$; (2) $\omega=\sqrt{\dfrac{3k\varphi_0}{ml^2\sin 2\varphi_0}}$

(3) $F_{Cx}=-F_{Dx}=\dfrac{k\varphi_0}{2h},F_{Cy}=2mg$; (4) $\omega_0'=\sqrt{\dfrac{3k(1-2\cot 2\varphi_0)\varphi_0}{2ml^2}}$

4-12 $f=\dfrac{b}{2\pi}\sqrt{\dfrac{k_1k_2}{m(a^2k_1+b^2k_2)}}$

4-13 $\varphi_m=\theta_0\dfrac{r/R}{1-\left(\dfrac{\omega}{\omega_0}\right)^2}$, 式中 $\omega_0=\dfrac{R}{\rho}\sqrt{\dfrac{2k}{m}}$

4-14 (1) $k_0=\dfrac{3}{4}mgr$; (2) $\omega_0=\sqrt{\dfrac{2g}{r}}$

4-15 $f=\dfrac{1}{2\pi}\sqrt{\dfrac{ag}{\rho^2+(r-a)^2}}$

4-16 $f_0=0.184\ \mathrm{Hz},\zeta=0.289,f_d=0.176\ \mathrm{Hz},T_d=5.677\ \mathrm{s}$

4-17 $c=\dfrac{2\pi m}{sT_1T_2}\sqrt{T_2^2-T_1^2}$

4-18 (1) $\eta=3.162,\ \Lambda=1.151$

(2) $\delta=1.456\ \mathrm{s}^{-1},\ T_d=0.79\ \mathrm{s}$;(3)$c_{cr}=39\ 626\ \mathrm{N\cdot s/m}$

4-19 $v_{cr}=96\ \mathrm{km/h}$

4-20 (1) $y=\dfrac{kdl^2}{kl^2-m\pi^2v^2}\sin\dfrac{\pi}{l}vt$; (2) $v_{cr}=\dfrac{1}{\pi}\sqrt{\dfrac{k}{m}}$

4-21 (1) $\omega=21.9\ \mathrm{rad/s}$; (2) $b=8.4\times10^{-3}\ \mathrm{mm}$

4－22 $x = 39.2\sin 7t$ mm

4－23 $x = -0.233\sin 8\pi t$ mm

4－24 (a) $x = \dfrac{a}{\sqrt{(1-s^2)^2 + (2\zeta s)^2}}\sin(\omega t - \varphi)$

$\varphi = \arctan\dfrac{2\zeta s}{1-s^2}$，其中 $s = \dfrac{\omega}{\omega_0}$，$\omega_0 = \sqrt{\dfrac{k}{m}}$，$\zeta = \dfrac{c}{2\sqrt{mk}}$

(b) $x' = \dfrac{\frac{c\omega a}{k}}{\sqrt{(1-s^2)^2 + (2\zeta s)^2}}\cos(\omega t - \varphi)$

4－25 $c = 107.6$ N·s/m

4－26 $k \leqslant 8.97$ kN/m

4－27 $(\ddot{x})_{max} = 84$ m/s^2

4－28 $k = 323$ kN/m

4－29 $\omega_0 = \sqrt{\dfrac{kd^2 - mgl}{ml^2}}$，$\varphi = \dfrac{kbd}{ml^2(\omega_0^2 - \omega^2)}\sin\omega t$

4－30 $\delta = \dfrac{e\left(\dfrac{\omega}{\omega_0}\right)^2}{1 - \left(\dfrac{\omega}{\omega_0}\right)^2}$，其中 $\omega_0 = \sqrt{\dfrac{k}{m}}$

4－31 $\omega_1^2 = 0.642\dfrac{k_2}{m_2}$，$\omega_2^2 = 1.558\dfrac{k_2}{m_2}$，$\gamma^{(1)} = 0.358$，$\gamma^{(2)} = -0.558$

4－32 $\omega_1^2 = \dfrac{k}{m}$，$\omega_2^2 = 3\dfrac{k}{m}$，$\gamma^{(1)} = 1$，$\gamma^{(2)} = -1$

4－33 $\omega_1 = 0.618\sqrt{\dfrac{k_t}{J}}$，$\omega_2 = 1.618\sqrt{\dfrac{k_t}{J}}$

4－34 $(m_A + m_B)\ddot{x} + m_B\ddot{\varphi} + 2kx = 0$，$\ddot{x} + l\ddot{\varphi} + g\varphi = 0$

$$\omega_{1,2} = \frac{(m_A + m_B)g + 2kl}{2m_A l} \mp \sqrt{\left[\frac{(m_A + m_B)g + 2kl}{2m_A l}\right]^2 - \frac{2kg}{m_A l}}$$

4－35 $\omega_1 = 0.342\sqrt{\dfrac{k}{m}}$，$\omega_2 = 1.46\sqrt{\dfrac{k}{m}}$

4－36 $\omega_1 = 2.33$ rad/s，$\omega_2 = 3.77$ rad/s

4－37 $\varphi_1 = \dfrac{k_t M_0}{J^2}\dfrac{1}{\omega_0^4 - 3\omega_0^2 p^2 + p^4}\sin pt$

$\varphi_2 = \dfrac{(2k_t - Jp^2)M_0}{J^2}\cdot\dfrac{1}{\omega_0^4 - 3\omega_0^2 p^2 + p^4}\sin pt$

式中 $\omega_0^2 = \dfrac{k_t}{J}$

第 五 章

5－1 $n_3 = -2n_O = 60$ r/min(负号表示与 n_O 转向相反)

5－2 $v_M=\sqrt{10}R\omega_O$；$a_M=R\sqrt{10(\alpha_O^2+\omega_O^4)-12\omega_O^2\cdot\alpha_0}$

5－3 $\omega_{\mathrm{I}}=2\omega\left(1+\dfrac{r_2}{r_1}\right)$，$\omega_{\mathrm{IV}}=\omega\dfrac{(r_1+r_2)(r_2+r_3)}{r_2(r_1+r_2-r_3)}$

5－4 $i_{4\mathrm{H}}=\dfrac{1}{11}$

5－5 $\omega_3=7$ rad/s；$\omega_{4\mathrm{r}}=5$ rad/s

5－6 $\omega'=\dfrac{\cos(\beta+2\theta)}{\sin 2\theta}\omega$，$\alpha'=\dfrac{\cos^2(\beta+2\theta)}{\sin^2 2\theta}\cdot\dfrac{\sin\theta\cos\beta}{\cos(\beta+\theta)}\omega^2$

5－7 $\omega_{\mathrm{r}}=1.047$ rad/s，$\omega=0.907$ rad/s

5－8 $\omega=\sqrt{\left(\dfrac{\pi n}{30}\right)^2+\omega_1^2+2\left(\dfrac{\pi n}{30}\right)\omega_1\cos\theta}$，$\alpha=\omega_1\dfrac{n\pi}{30}\sin\theta$

5－9 $\omega=5.82$ rad/s，与轴 x 和轴 z 的正方向间所夹的角各为 $\theta=30°59'$ 和 $\beta=59°1'$；$\alpha=15$ rad/s^2，方向沿轴 y。

5－10 $v_A=0.55$ m/s，$a_A=0.054$ m/s^2

5－11 $\boldsymbol{\alpha}=0.125\boldsymbol{i}$ rad/s^2，$\boldsymbol{a}_A=(0.094\boldsymbol{i}-0.73\boldsymbol{j}-0.033\boldsymbol{k})$ m/s^2

5－12 (1) $v_B=0.817$ m/s，$a_B=0.37$ m/s^2

(2) $v_B=1.171$ m/s，$a_B=1.999$ m/s^2

(3) $v_B=0.2$ m/s，$a_B=5.271$ m/s^2

5－13 $\boldsymbol{v}_A=-0.689\boldsymbol{i}$ m/s，$\boldsymbol{a}_A=(4.652\boldsymbol{j}-6.651\boldsymbol{k})$ m/s^2

$\boldsymbol{\omega}=(6.928\boldsymbol{j}+7\boldsymbol{k})$ rad/s，$\boldsymbol{\alpha}=-20.78\boldsymbol{i}$ rad/s^2

5－14 $v_C=0$，$v_B=0.4$ m/s，$a_C=0.4$ m/s^2，$a_B=0.4\sqrt{5}$ m/s^2

5－15 $\omega=0.24$ rad/s，逆时针转向转动。

5－16 $F_E=F_F=\dfrac{\sqrt{2}mr^2}{4l}\omega\omega_{\mathrm{e}}$

5－17 $M_{\mathrm{g}}=5\,760$ N·m；$F=9.6$ kN

5－18 $M_{\mathrm{gmax}}=27.91$ kN·m；$F_{A\max}=F_{B\max}=14.69$ kN

第 六 章

6－1 $\ddot{\varphi}+\dfrac{\beta}{m(t)}\dot{\varphi}+\dfrac{g}{l}\sin\varphi=0$

6－2 $\ddot{x}=-g-\dfrac{\dot{f}(t)}{f(t)}v_{\mathrm{r}}-\dfrac{F_{\mathrm{R}}(x,\dot{x})}{m_0 f(t)}$

6－3 $F(t)=\rho v^2+\rho vgt$；$F_{\mathrm{N}}(t)=(l-vt)\rho g$

6－4 $v=\dfrac{cv_{\mathrm{r}}}{f}\left[1-\left(\dfrac{m_0-ct}{m_0}\right)^{\frac{f}{c}}\right]$

6－5 $m=m_0\mathrm{e}^{-\frac{a}{v_{\mathrm{r}}}t}$

6－6 $t=0$ s，$v_0=0$，$a_0=11.2$ m/s^2

$t=45$ s，$v_{45}=814.5$ m/s，$a_{45}=28.38$ m/s^2

$t = 90$ s，　$v_{90} = 3\ 953$ m/s，　$a_{90} = 200.2$ m/s^2

6－7　$m_1/m_2 = 3.32$

6－8　$F = qv$

6－9　$v = 230$ m/s

6－10　$q = 49.66$ kg/s

6－11　(1) 26 460 N，　(2) 230.3 m/s