4-2 解题过程：

（1）由 $f(t)=\sin wt\left[u(t)-u(t-T/2)\right]=\sin wtu(t)+\sin\left[w(t-T/2)\right]u(t-T/2)$ 得

$$\mathscr{L}\left[f(t)\right]=\mathscr{L}\left[\sin wtu(t)\right]+\mathscr{L}\left\{\sin\left[w(t-T/2)\right]u(t-T/2)\right\}$$
$$=\frac{\omega^2}{s^2+\omega^2}+\frac{\omega}{s^2+\omega^2}e^{-\frac{sT}{2}}$$
$$=\frac{\omega}{s^2+\omega^2}\left(1+e^{-\frac{sT}{2}}\right)$$

（2）由 $\sin(wt+\varphi)=\sin wt\cos\varphi+\cos wt\sin\varphi$ 得

$$\mathscr{L}\left[\sin(wt+\varphi)\right]=\mathscr{L}\left(\sin wt\cos\varphi\right)+\mathscr{L}\left(\cos wt\sin\varphi\right)$$
$$=\frac{\omega\cos\varphi}{s^2+\omega^2}+\frac{s\sin\varphi}{s^2+\omega^2}$$
$$=\frac{\omega\cos\varphi+s\sin\varphi}{s^2+\omega^2}$$

4-3 解题过程：

（1）由 $f(t)=e^{-(t-2)}u(t-2)\cdot e^{-2}$ 得

$$\mathscr{L}\left[f(t)\right]=e^{-2}\mathscr{L}\left[e^{-(t-2)}u(t-2)\right]$$
$$=e^{-2}\cdot\frac{1}{s+1}\cdot e^{-2s}$$
$$=\frac{1}{s+1}e^{-2(s+1)}$$

（2）$\mathscr{L}\left[f(t)\right]=\dfrac{1}{s+1}e^{-2s}$

（3）由 $f(t)=e^{-t}u(t)\cdot e^{2}$ 得

$$\mathscr{L}\left[f(t)\right]=e^{2}\mathscr{L}\left[e^{-t}u(t)\right]=\frac{e^2}{s+1}$$

（4）由 $f(t)=\sin\left[2(t-1)+2\right]u(t-1)$

$$=\cos 2\sin\left[2(t-1)\right]u(t-1)+\sin 2\cos\left[2(t-1)\right]u(t-1)$$ 得

$$\begin{aligned}\mathscr{L}\left[f(t)\right]&=\cos 2\mathscr{L}\left\{\sin\left[2(t-1)\right]u(t-1)\right\}+\sin 2\mathscr{L}\left\{\sin\left[2(t-1)\right]u(t-1)\right\}\\&=\frac{2\cos 2}{s^2+4}e^{-s}+\frac{s\sin 2}{s^2+4}e^{-s}\\&=\frac{2\cos 2+s\sin 2}{s^2+4}e^{-s}\end{aligned}$$

（5）由 $f(t)=(t-1)u(t-1)-(t-2)u(t-2)-u(t-2)$ 得

$$\begin{aligned}\mathscr{L}\left[f(t)\right]&=\mathscr{L}\left[(t-1)u(t-1)\right]-\mathscr{L}\left[(t-2)u(t-2)\right]-\mathscr{L}\left[u(t-2)\right]\\&=\frac{1}{s^2}e^{-s}-\frac{1}{s^2}e^{-2s}-\frac{1}{s}e^{-s}\\&=\frac{1}{s^2}\left[1-(1+s)e^{-s}\right]\end{aligned}$$

4-4 解题过程：

（1）$\mathscr{L}^{-1}\left(\frac{1}{s+1}\right)=e^{-t}$

（2）由 $\frac{4}{2s+3}=\frac{2}{s+\frac{3}{2}}$ 得 $\mathscr{L}^{-1}\left[\frac{4}{2s+3}\right]=\mathscr{L}^{-1}\left[\frac{2}{s+\frac{3}{2}}\right]=2e^{-\frac{3}{2}t}$

（3）由 $\frac{4}{s(2s+3)}=\frac{4}{3}\left(\frac{1}{s}-\frac{1}{s+\frac{3}{2}}\right)$ 得 $\mathscr{L}^{-1}\left[\frac{4}{s(2s+3)}\right]=\frac{4}{3}\mathscr{L}^{-1}\left[\frac{1}{s}\right]-\frac{4}{3}\mathscr{L}^{-1}\left[\frac{1}{s+\frac{3}{2}}\right]$

（4）由 $\frac{1}{s(s^2+5)}=\frac{1}{5}\left(\frac{1}{s}-\frac{s}{s^2+5}\right)$ 得

$$\mathscr{L}^{-1}\left[\frac{1}{s(s^2+5)}\right]=\frac{1}{5}\mathscr{L}^{-1}\left[\frac{1}{s}\right]-\frac{1}{5}\mathscr{L}^{-1}\left[\frac{s}{s^2+5}\right]=\frac{1}{5}\left(1-\cos\sqrt{5}t\right)$$

（5）由 $\frac{3}{(s+4)(s+2)}=\frac{3}{2}\left(\frac{1}{s+2}-\frac{1}{s+4}\right)$ 得

$$\mathscr{L}^{-1}\left[\frac{3}{(s+4)(s+2)}\right]=\frac{3}{2}\mathscr{L}^{-1}\left[\frac{1}{s+2}\right]-\frac{3}{2}\mathscr{L}^{-1}\left[\frac{1}{s+4}\right]=\frac{3}{2}\left(e^{-2t}-e^{-4t}\right)$$

（6）由 $\frac{3s}{(s+4)(s+2)}=\frac{6}{s+4}-\frac{3}{s+2}$ 得

$$\mathscr{L}^{-1}\left[\frac{3s}{(s+4)(s+2)}\right]=\mathscr{L}^{-1}\left[\frac{6}{s+4}\right]-\mathscr{L}^{-1}\left[\frac{3}{s+2}\right]=6e^{-4t}-3e^{-2t}$$

（7）$\mathscr{L}^{-1}\left[\frac{1}{s^2+1}+1\right]=\sin t+\delta(t)$

（8）由 $\frac{1}{s^2-3s+2}=\frac{1}{s-2}-\frac{1}{s-1}$ 得

$$\mathscr{L}^{-1}\left[\frac{1}{s^2-3s+2}\right]=\mathscr{L}^{-1}\left(\frac{1}{s-2}\right)-\mathscr{L}^{-1}\left(\frac{1}{s-1}\right)=e^{2t}-e^{t}$$

（9）由 $\frac{1}{s(RCs+1)}=\frac{1}{s}-\frac{1}{s+\frac{1}{RC}}$ 得 $\mathscr{L}^{-1}\left[\frac{1}{s(RCs+1)}\right]=1-e^{-\frac{t}{RC}}$

（10）由 $\frac{1-RCs}{s(RCs+1)}=\frac{1}{s}-\frac{2}{s+\frac{1}{RC}}$ 得 $\mathscr{L}^{-1}\left[\frac{1-RCs}{s(RCs+1)}\right]=1-2e^{-\frac{t}{RC}}$

（11）由 $\frac{w}{s^2+w^2}\cdot\frac{1}{(RCs+1)}=\frac{RCw}{1+(RCw)^2}\left(\frac{1}{s+\frac{1}{RC}}-\frac{s}{s^2+w^2}+\frac{\frac{1}{RCw}\cdot w}{s+\frac{1}{RC}}\right)$ 得

$$\mathscr{L}^{-1}\left[\frac{w}{s^2+w^2}\cdot\frac{1}{(RCs+1)}\right]=\frac{RCw}{1+(RCw)^2}\left[e^{-\frac{t}{RC}}-\cos wt+\frac{1}{RCw}\sin wt\right]$$

（12）由 $\frac{4s+5}{s^2+5s+6}=\frac{7}{s+3}-\frac{3}{s+2}$ 得 $\mathscr{L}^{-1}\left[\frac{4s+5}{s^2+5s+6}\right]=7e^{-3t}-3e^{-2t}$

（13）由 $\frac{100(s+50)}{s^2+201s+200}=\frac{100(s+50)}{(s+1)(s+200)}$ 得

$$\mathscr{L}^{-1}\left[\frac{100(s+50)}{s^2+201s+200}\right]=\frac{100}{199}\left(49e^{-t}+150e^{-200t}\right)$$

（14）令 $\frac{s+3}{(s+1)^3(s+2)}=\frac{k_1}{s+2}+\frac{k_2}{(s+1)^3}+\frac{k_3}{(s+1)^2}+\frac{k_4}{s+1}$

则 $k_1=\left.\frac{s+3}{(s+1)^3}\right|_{s=-2}=-1$，$k_2=\left.\frac{s+3}{s+1}\right|_{s=-1}=2$，

$k_3=\left.\frac{d}{ds}\left(\frac{s+3}{s+1}\right)\right|_{s=-1}=-1$，　$k_4=\left.\frac{d^2}{ds^2}\left(\frac{s+3}{s+1}\right)\right|_{s=-1}=1$

从而 $\dfrac{s+3}{(s+1)^3(s+2)}=\dfrac{-1}{s+2}+\dfrac{2}{(s+1)^3}-\dfrac{1}{(s+1)^2}+\dfrac{1}{s+1}$

所以 $\mathscr{L}^{-1}\left[\dfrac{s+3}{(s+1)^3(s+2)}\right]=-e^{-t}+\left(t^2-t+1\right)e^{-t}$

（15）由 $\dfrac{A}{s^2+K^2}=\dfrac{A}{K}\cdot\dfrac{K}{s^2+K^2}$ 得 $\mathscr{L}^{-1}\left[\dfrac{A}{s^2+K^2}\right]=\dfrac{A}{K}\sin Kt$

（16）由于 $\mathscr{L}^{-1}\left[\dfrac{s}{\left(s^2+3\right)^2}\right]=\dfrac{1}{2\sqrt{3}}t\sin Kt$ 由拉氏变换的积分性质可得

$$\mathscr{L}^{-1}\left[\frac{1}{\left(s^2+3\right)^2}\right]=\int_0^t\frac{1}{2\sqrt{3}}\tau\sin\left(\sqrt{3}\tau\right)d\tau=\frac{\sqrt{3}}{18}\sin\left(\sqrt{3}t\right)-\frac{t}{6}\cos\left(\sqrt{3}t\right)$$

4-19 解题过程：

由于 $f(t)$ 可以写作 $f(t)=\sum\limits_{k=0}^{\infty}f_1(t-kT)$

$$=F_1(s)\sum_{k=0}^{\infty}e^{-skT}=\frac{F_1(s)}{1-e^{-sT}}$$

则 $\mathscr{L}\left[f(t)\right]=F(s)=\mathscr{L}\left[\sum\limits_{k=0}^{\infty}f_1(t-kT)\right]$

$$=\sum_{k=0}^{\infty}\mathscr{L}\left[f_1(t-kT)\right]=\sum_{k=0}^{\infty}F_1(s)e^{-skT}$$

$$=F_1(s)\sum_{k=0}^{\infty}e^{-skT}=\frac{F_1(s)}{1-e^{-sT}}$$

4-20 解题过程：

（1）周期矩形脉冲信号的第一个周期时间信号为 $f_1(t)=u(t)-u\left(t-\dfrac{T}{2}\right)$

所以 $F_1(s)=\dfrac{1}{s}\left(1-e^{-\frac{T}{2}s}\right)$ 则 $F(s)=\dfrac{F_1(s)}{1-e^{-sT}}=\dfrac{1-e^{-s\frac{T}{2}}}{s\left(1-e^{-sT}\right)}=\dfrac{1}{s\left(1+e^{-s\frac{T}{2}}\right)}$

（2）正弦全波整流脉冲信号第一周期时间信号为

$$f_1(t)=\sin(wt)\left[u(t)-u\left(t-\frac{T}{2}\right)\right]=\sin wtu(t)+\sin\left[w\left(t-\frac{T}{2}\right)\right]u\left(t-\frac{T}{2}\right)$$

所以 $F_1(s)=\frac{w}{s^2+w^2}+\frac{w}{s^2+w^2}e^{-\frac{T}{2}s}$ 则 $F(s)=\frac{F_1(s)}{1-e^{-\frac{T}{2}s}}=\frac{w}{s^2+w^2}\cdot\frac{1+e^{-\frac{T}{2}s}}{1-e^{-\frac{T}{2}s}}$

4-27 解题过程：由 $e(t)=e^{-t}$ 得 $E(s)=\mathscr{L}\left[e(t)\right]=\frac{1}{s+1}$

$$r_{zs}(t)=r(t)=\frac{1}{2}e^{-t}-e^{-2t}+2e^{-3t}$$

$$R_{zs}(s)=\mathscr{L}\left[r_{zs}(t)\right]=\frac{1}{2(s+1)}-\frac{1}{s+2}+\frac{2}{s-3}$$

故 $H(s)=\frac{R_{zs}(s)}{E(s)}$

$$=\left[\frac{1}{2(s+1)}-\frac{1}{s+2}+\frac{2}{s-3}\right]\cdot(s+1)$$

$$=\frac{1}{2}-\frac{s+1}{s+2}+\frac{2(s+1)}{s-3}$$

$$=\frac{3}{2}+\frac{1}{s+2}-\frac{8}{s-3}$$

所以 $h(s)=\mathscr{L}^{-1}\left[H(s)\right]=\frac{3}{2}\delta(t)+\left(e^{-2t}+8e^{3t}\right)u(t)$

4-35 解题过程：

$$H(s)=K\frac{\prod_{i=1}^{k}(s-z_i)}{\prod_{j=1}^{l}(s-p_j)}$$ （K 为系数）

$$=K\frac{s(s+2-j)(s+2+j)}{(s+3)(s+1-3j)(s+1+3j)}$$

$$=K\frac{s\left(s^2+4s+5\right)}{(s+3)\left(s^2+2s+10\right)}$$

又知 $H(\infty)=5$，即 $\lim_{s\to\infty}H(s)=K=5$

$$H(s)=\frac{5s\left(s^2+4s+5\right)}{(s+3)\left(s^2+2s+10\right)}=\frac{5\left(s^3+4s^2+5s\right)}{s^3+5s^2+16s+30}$$

4-38 解题过程：

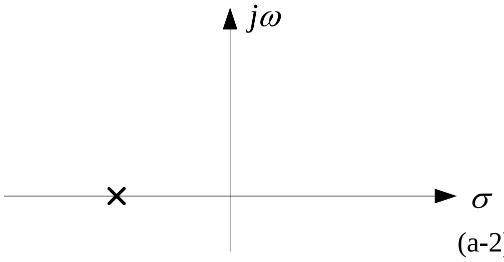
分别画出题图对应的零极点图如下图(a-2)~(f-2)所示。

（1）由解图(a-1)有，

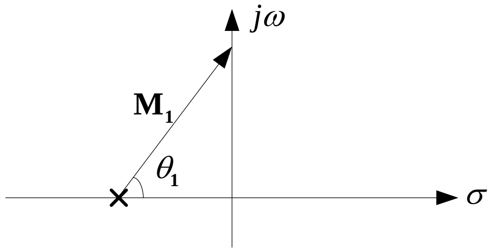
当$\omega=0$时，极点矢量$\mathbf{M_1}$最短，辐角$\theta_1=0$，随着$\omega\uparrow$，有$\mathbf{M_1}\uparrow$，$\theta_1\uparrow$

当$\omega\to\infty$时，$\mathbf{M_1}\to\infty$，$\theta_1\to\dfrac{\pi}{2}$

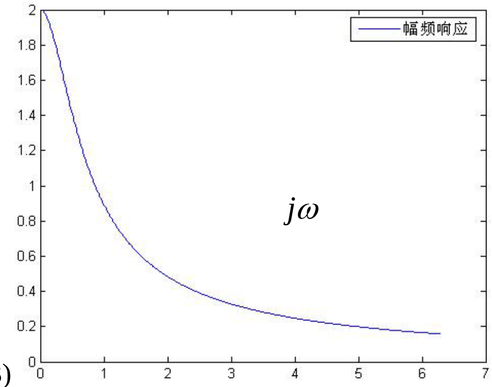
幅频、相频特性如图(a-3)、(a-4)(极点选取-0.5为例)



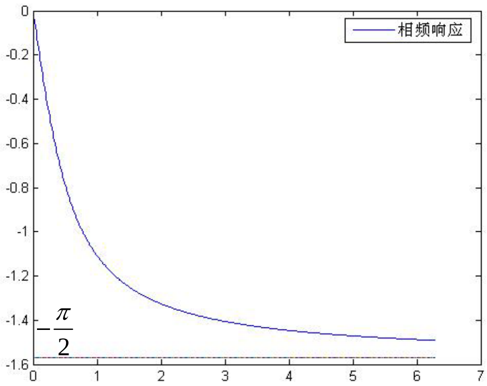
(a-1)



(a-2)



(a-3)



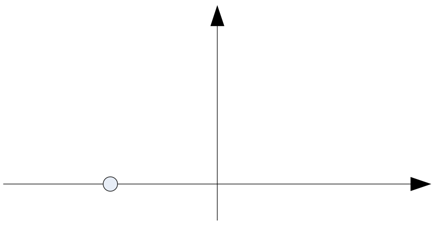
(a-4)

（2）由解图(b)有，

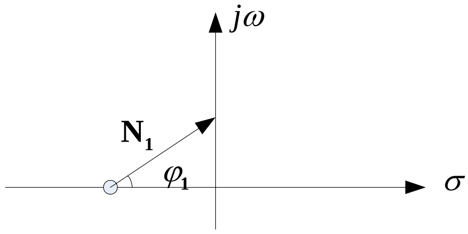
当$\omega=0$时，$\mathbf{N_1}$最短，辐角$\varphi_1=0$，随着$\omega\uparrow$，有$\mathbf{N_1}\uparrow$，$\varphi_1\uparrow$

当$\omega\to\infty$时，$\mathbf{N_1}\to\infty$，$\varphi_1\to\dfrac{\pi}{2}$

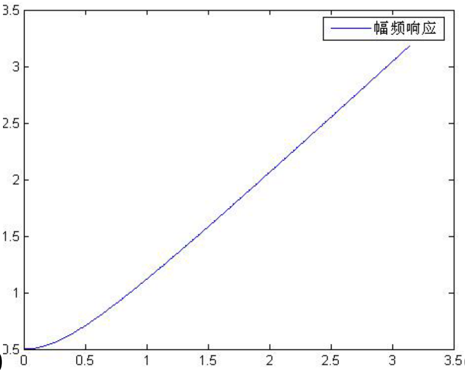
幅频、相频特性如图(以零点为-0.5为例)

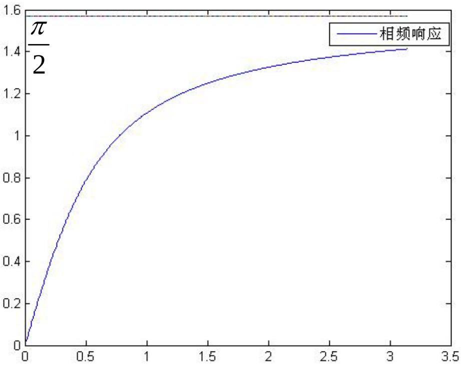


(b-1)



(b-2)

(b-3)


(b-4)


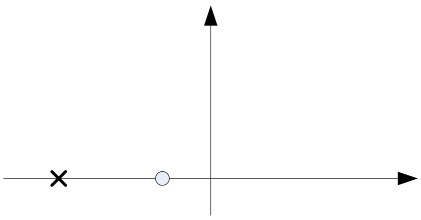
（3）由解图(b)有，

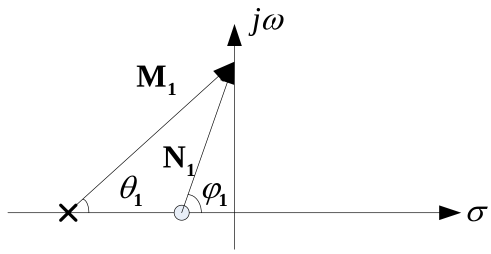
$\omega=0$ 时，$\mathbf{M_1}$，$\mathbf{N_1}$ 均为最短，辐角 $\theta_1=\varphi_1=0$

$\omega\uparrow$，则有 $\mathbf{M_1}\uparrow$，$\mathbf{N_1}\uparrow$，且有 $\theta_1\uparrow$，$\varphi_1\uparrow$，且有 $\theta_1<\varphi_1$

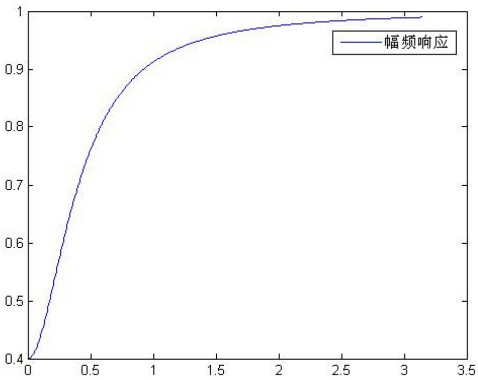
$\omega\to\infty$ 时，$\mathbf{M_1}\to\infty$，$\mathbf{N_1}\to\infty$，$\theta_1\to\frac{\pi}{2}$，$\varphi_1\to\frac{\pi}{2}$

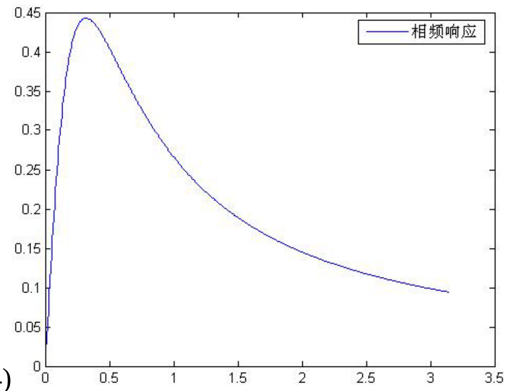
幅频、相频特性如图(以极点-0.5，零点-0.2 为例)


(c-1)



(c-2)

(c-3)


(c-4)


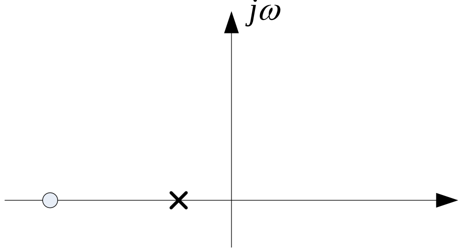
（4）由解图(d)有，

$\omega=0$ 时，$\mathbf{M_1}$，$\mathbf{N_1}$ 均为最短，辐角 $\theta_1=\varphi_1=0$

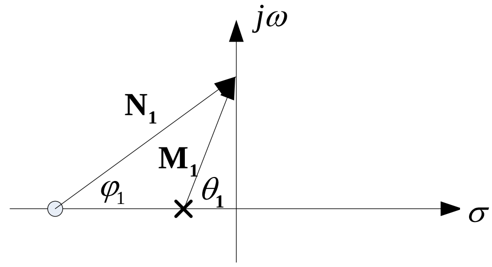
$\omega\uparrow$，则有 $\mathbf{M_1}\uparrow$，$\mathbf{N_1}\uparrow$，且有 $\mathbf{M_1}>\mathbf{N_1}$；$\theta_1\uparrow$，$\varphi_1\uparrow$，且有 $\theta_1>\varphi_1$

$\omega\to\infty$ 时，$\mathbf{M_1}\to\infty$，$\mathbf{N_1}\to\infty$，$\theta_1\to\frac{\pi}{2}$，$\varphi_1\to\frac{\pi}{2}$

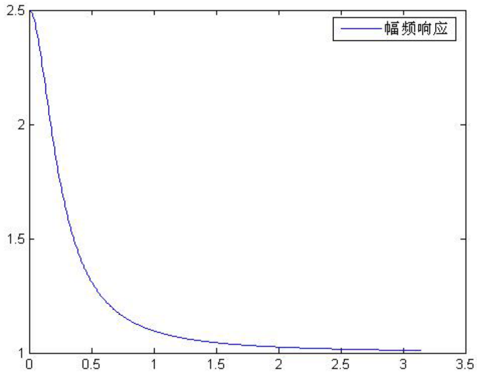
幅频、相频特性如图(以零点-0.5，极点-0.2 为例)



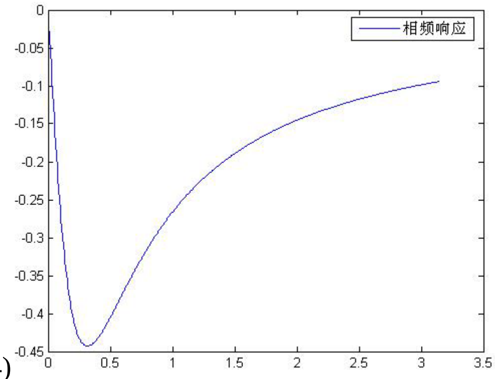
(d-1)



(d-2)



(d-3)



(d-4)

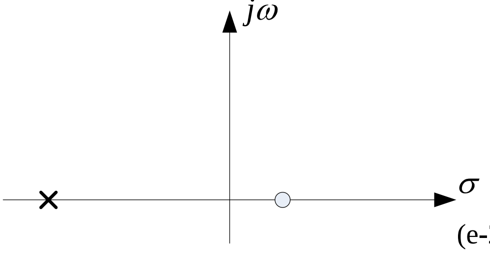
（5）由解图(e)有，

$\omega = 0$ 时，$\mathbf{M}_1$，$\mathbf{N}_1$ 均为最短，辐角 $\theta_1 = 0$，$\varphi_1 = \pi$

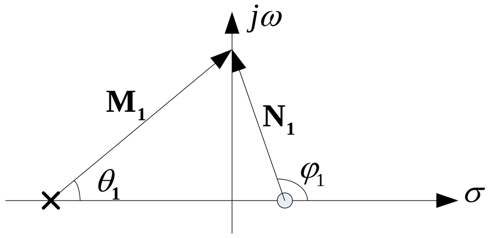
$\omega\uparrow$，则有 $\mathbf{M}_1\uparrow$，$\mathbf{N}_1\uparrow$，$\mathbf{M}_1 > \mathbf{N}_1$；$\theta_1\uparrow$，$\varphi_1\downarrow$

$\omega \to \infty$ 时，$\mathbf{M}_1 \to \infty$，$\mathbf{N}_1 \to \infty$，$\theta_1 \to \frac{\pi}{2}$，$\varphi_1 \to \frac{\pi}{2}$

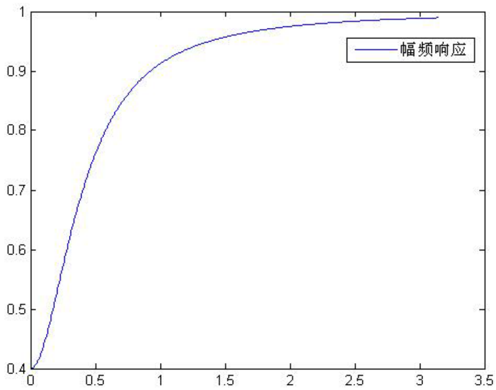
幅频、相频特性如图(以极点-0.5，零点 0.2 为例)



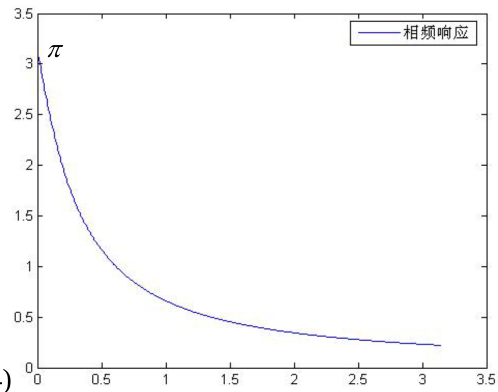
(e-1)



(e-2)



(e-3)



(e-4)

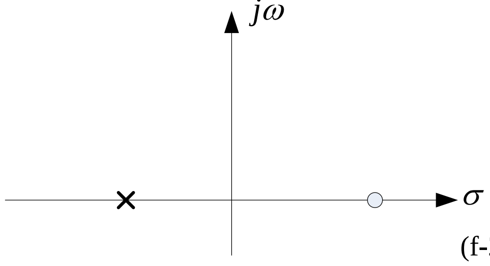
（6）由解图(e)有，

$\omega = 0$ 时，$\mathbf{M}_1$，$\mathbf{N}_1$ 均为最短，辐角 $\theta_1 = 0$，$\varphi_1 = \pi$

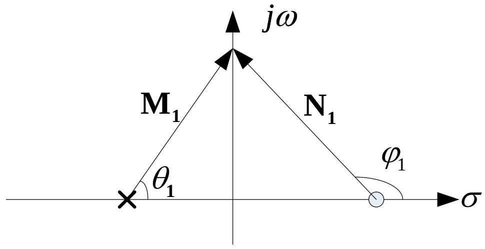
$\omega\uparrow$，则有$\mathbf{M_1}\uparrow$，$\mathbf{N_1}\uparrow$，$\mathbf{M_1}>\mathbf{N_1}$；$\theta_1\uparrow$，$\varphi_1\downarrow$，但相对关系与(e)中不同。

$\omega\rightarrow\infty$时，$\mathbf{M_1}\rightarrow\infty$，$\mathbf{N_1}\rightarrow\infty$，$\theta_1\rightarrow\dfrac{\pi}{2}$，$\varphi_1\rightarrow\dfrac{\pi}{2}$

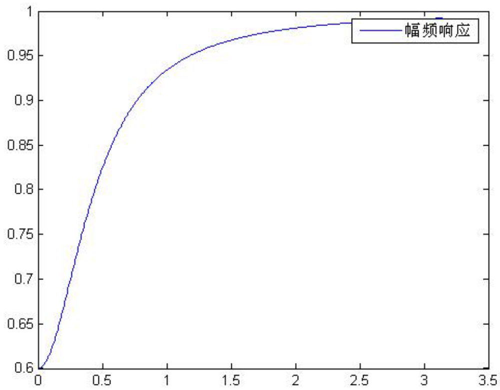
幅频、相频特性如图(以极点-0.5，零点 0.3 为例)



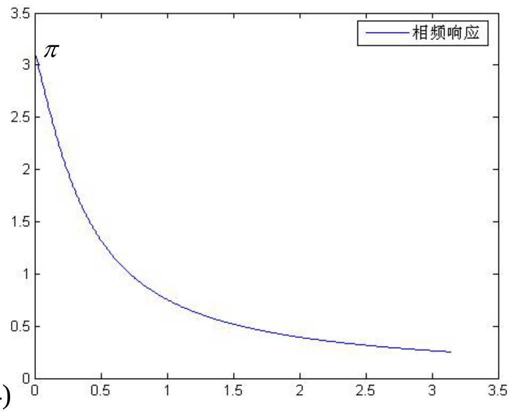
(f-1)



(f-2)



(f-3)



(f-4)