

、 答案、学长笔记、辅导班课程，访问：W

中国科学院 2012 年数学分析真题解析

1.(15×2 分) 计算极限

$$(1) \lim_{n \rightarrow \infty} n^3 \left(2 \sin \frac{1}{n} - \sin \frac{2}{n} \right)$$

$$(2) \lim_{x \rightarrow \infty} \left(\sqrt{\cos \frac{1}{x^2}} \right)^{x^4}$$

【解答】

(1)

$$\lim_{n \rightarrow \infty} n^3 \left(2 \sin \frac{1}{n} - \sin \frac{2}{n} \right) = \lim_{n \rightarrow \infty} n^3 \left\{ 2 \left[\left(\frac{1}{n} - \frac{1}{6n^3} \right) + o\left(\frac{1}{n^3}\right) \right] - \left[\frac{2}{n} - \frac{1}{6} \left(\frac{2}{n} \right)^3 + o\left(\frac{1}{n^3}\right) \right] \right\} = \lim_{n \rightarrow \infty} \frac{n^3}{n^3} = 1$$

$$(2) \left(\sqrt{\cos \frac{1}{x^2}} \right)^{x^4} = \left(\cos \frac{1}{x^2} \right)^{\frac{x^4}{2}}, \lim_{x \rightarrow \infty} \left(\cos \frac{1}{x^2} \right)^{\frac{x^4}{2}} = \lim_{x \rightarrow \infty} e^{\frac{1}{2} \ln \left(\cos \frac{1}{x^2} \right)^{x^4}} = e^{\frac{1}{2} \lim_{x \rightarrow \infty} \frac{\ln \left(\cos \frac{1}{x^2} \right)^{x^4}}{x^4}},$$