

## 中国科学院2002年高等数学丙真题解析

一. 填空题(本题共5小题, 每小题3分, 满分15分。把答案填在题中横线上)

1.  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 2x} - \sqrt{x^2 - 1}) = \underline{\hspace{2cm}}$

**【解答】**

$$\begin{aligned} & \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 2x} - \sqrt{x^2 - 1}) \\ &= \lim_{x \rightarrow +\infty} [\sqrt{(x+1)^2 - 1} - \sqrt{x^2 - 1}] = \lim_{x \rightarrow +\infty} \left\{ \sqrt{(x+1)^2 - 1} - (x+1) \right\} + (x+1) + (x - \sqrt{x^2 - 1}) - x \Big\} = 1 \end{aligned}$$

2.  $\int_1^4 \frac{dx}{1 + \sqrt{x}} = \underline{\hspace{2cm}}$

**【解答】**

$$\int_1^4 \frac{dx}{1 + \sqrt{x}} = 2 \int_1^2 \frac{y dy}{1 + y} = 2 \int_1^2 \left( \frac{1}{1 + y} + \frac{y}{1 + y} \right) dy = 2 \left( \ln \frac{3}{2} + \frac{3}{2} - 2 \right) = 3 - 2 \ln \frac{3}{2}$$