

解：1) $\Psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} & 0 \leq x \leq a \\ 0 & \text{other} \end{cases}$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2 \mu a^2}, n = 1, 2, 3, \dots$$

2) 第二激发态 $\Psi_3(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin \frac{3\pi x}{a} & 0 \leq x \leq a \\ 0 & \text{other} \end{cases}$

概率密度 $P_1 = |\Psi_3|^2$

第三激发态 $\Psi_4(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin \frac{4\pi x}{a} & 0 \leq x \leq a \\ 0 & \text{other} \end{cases}$

概率密度 $P_2 = |\Psi_4|^2$

示意图略！

$$3) x_{mn} = \int_{-\infty}^{+\infty} \Psi_m^*(x) x \Psi_n(x) dx = \int_0^a x \Psi_m(x) \Psi_n(x) dx$$

当 $m \neq n$ 时