中科大 2016年量子力学 真题答案

1. $E_n = \frac{n^2\hbar^2}{2I}$ $\psi_n = \frac{1}{\sqrt{2\pi}} \cdot e^{in\varphi}$ $n=0, \pm1, \pm2 \cdots$

(1) $\psi(\varphi) = A \cdot \left[\psi_0 + \frac{1}{2}(\psi_1 + \psi_{-1}) + \psi_{-1}\right]$

$\psi(\varphi) = \frac{2}{\sqrt{14}} \cdot \psi_0 + \frac{1}{\sqrt{14}} \cdot \psi_1 + \frac{3}{\sqrt{14}} \cdot \psi_{-1}$

能量测值为 $E_0 = 0$ 的概率为 $\frac{2}{7}$

$E_1 = \frac{\hbar^2}{2I}$ 概率为 $\frac{5}{7}$

(2) $\psi(\varphi, t) = \frac{2}{\sqrt{14}} \psi_0 + \frac{1}{\sqrt{14}} \cdot \psi_1 \cdot e^{-\frac{iE_1 t}{\hbar}} + \frac{3}{\sqrt{14}} \cdot \psi_{-1} \cdot e^{-\frac{iE_{-1} t}{\hbar}}$

$= \frac{1}{\sqrt{28\pi}} \cdot \left(2 + e^{i\varphi - \frac{i\hbar t}{2I}} + 3e^{-i\varphi - \frac{i\hbar t}{2I}}\right)$

$\langle \varphi \rangle = \frac{1}{28\pi} \cdot \int_0^{2\pi} (2 + e^{-i\varphi + i\omega t} + 3e^{+i\varphi + i\omega t}) \varphi \cdot (2 + e^{i\varphi - i\omega t} + 3e^{-i\varphi - i\omega t}) \, d\varphi$

$= \frac{1}{28\pi} \int \Big[4\varphi + 2(e^{-i\varphi + i\omega t} + e^{i\varphi - i\omega t})\varphi + 2\cdot3(e^{i\varphi + i\omega t} + e^{-i\varphi - i\omega t})\varphi$

$+ \varphi \cdot (e^{-i\varphi} + 3e^{i\varphi}) \cdot (e^{i\varphi} + 3e^{-i\varphi}) \Big] d\varphi$

$= \frac{1}{28\pi} \left\{ 2\varphi^2 \Big|_0^{2\pi} + \int_0^{2\pi} \varphi \cdot (1 + 3 + 3e^{-2i\varphi} + 3e^{2i\varphi}) \, d\varphi + \int_0^{2\pi} [2\cdot2\cos(\varphi - \omega t) + 6\cdot2\cos(\varphi + \omega t)] \varphi \, d\varphi \right\}$

$= \frac{1}{28\pi} \left\{ 4\varphi^2 \Big|_0^{2\pi} + \int_0^{2\pi} 2\cdot3 \cdot \cos 2\varphi \cdot \varphi \cdot d\varphi + \int_0^{2\pi} [2\cdot2\cos(\varphi - \omega t) + 2\cdot6\cos(\varphi + \omega t)] \varphi \cdot d\varphi \right\}$

$= \frac{1}{28\pi} \left(4 \cdot 4\pi^2 + 4 \int_0^{2\pi} \varphi \cdot \cos(\varphi - \omega t) d\varphi + 12 \int_0^{2\pi} \varphi \cos(\varphi + \omega t) \, d\varphi \right)$

$= \frac{1}{28\pi} (16\pi^2 + 4 \cdot (2\pi) \cdot \sin\omega t + 12 \cdot 2\pi \sin\omega t)$