Theory of Machines and Mechanisms (机械原理)

Chapter 10
Balancing of Machinery

10.1 Purpose and Methods of Balancing

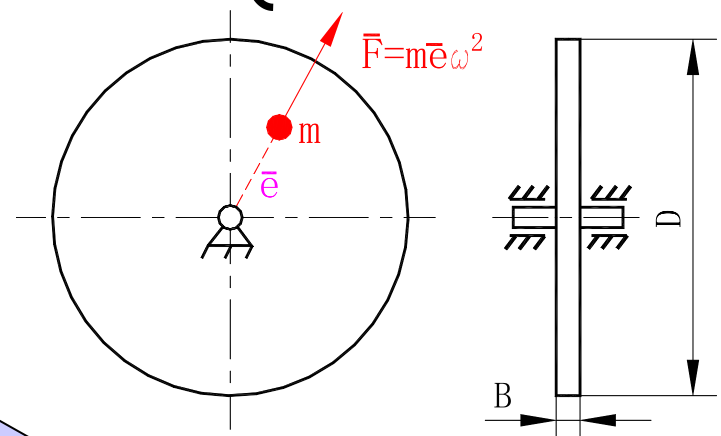
10.1.1 Purpose

Center of Mass

Rotating Center

→ Not Coincide

- Asymmetry 不对称
- Uneven Distribution 不均匀分布
- Errors 误差



The centrifugal离心力 —— inertia(惯性) force

$F = me\omega^2$

↓

vibration振动 → Resonant共振; 精度下降 可靠性下降 → Eliminate消除 unwanted Centrifugal Forces

However, some machines, e.g. vibrators（振动器）, shock drillers (冲击钻), etc. do work by vibration.

10.1.2 Methods

Balancing(平衡)——The process of designing or modifying machinery in order to reduce unwanted vibration to an acceptable level, and possibly to eliminate it entirely

((1))Balancing of rotor 转子的平衡

Parts constrained to rotate about a fixed axis

(a) Rigid rotors(刚性转子)

(b) Flexible rotors(柔性转子)

(2) Balancing of mechanisms 机构的平衡

——做复杂平面运动的构件在机架上的平衡

inertia forces 惯性力

inertia moments惯性力矩

10.2 Balancing of Disk-like Rotors

10.2.1 Conditions for the Balancing of a Disk-like Rotor

$\frac{d}{b} > 5$ 假设质量分布在同一回转面内

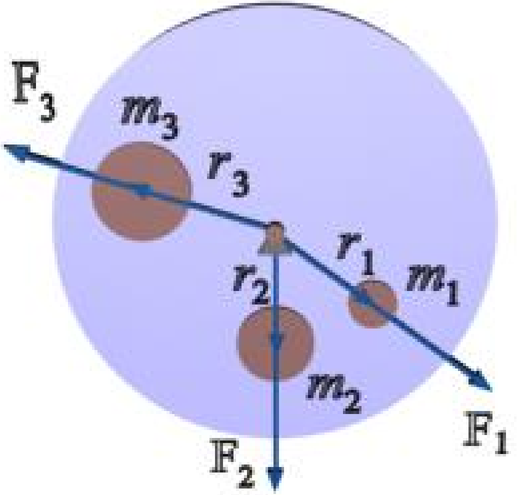
Balance(平衡)

The vectorial(矢量) sum of centrifugal (离心的) forces is zero. $\sum \overrightarrow{F_i} = 0$

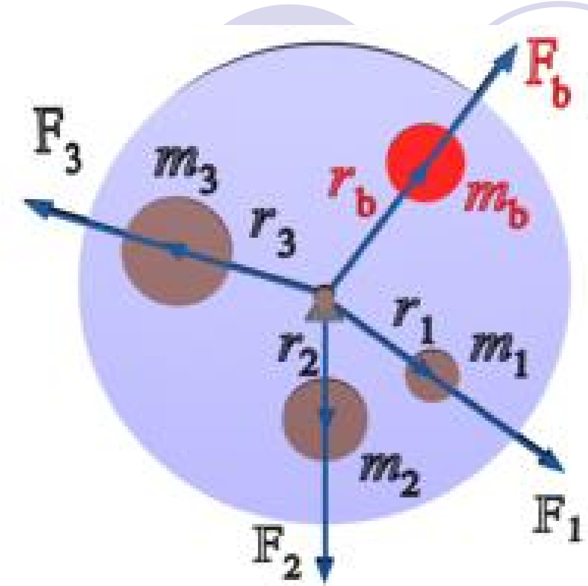
The mass centre of the system coincides with the shaft centre.

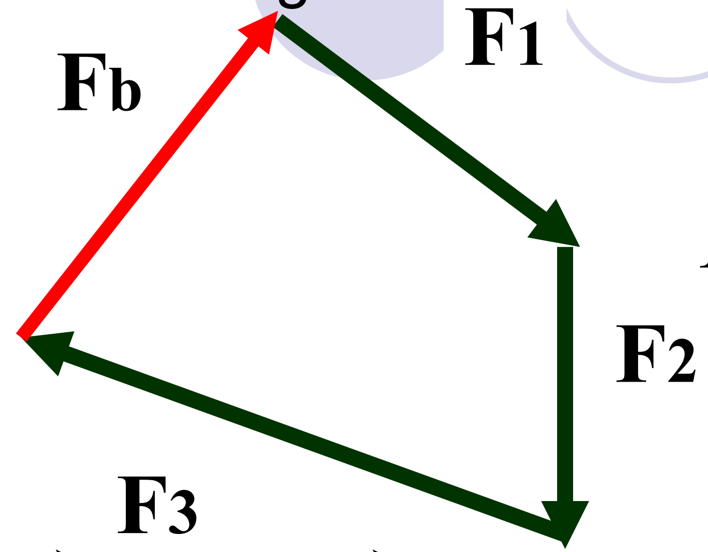
Imbalance(不平衡): $\sum \overrightarrow{F_i} \neq 0$

The mass centre of the system doesn't coincide with the shaft centre.



The counterweight(配重) is added to the system so that the velocity sum of centrifugal forces is zero.





$$F = mr\omega^2$$

$$m_1\vec{r_1}\cdot\omega^2 + m_2\vec{r_2}\cdot\omega^2 + m_3\vec{r_3}\cdot\omega^2 + m_b\vec{r_b}\cdot\omega^2 = 0$$

$$me = m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + m_b\vec{r_b} = 0$$ **e=0 (总质心在回转轴线上)**

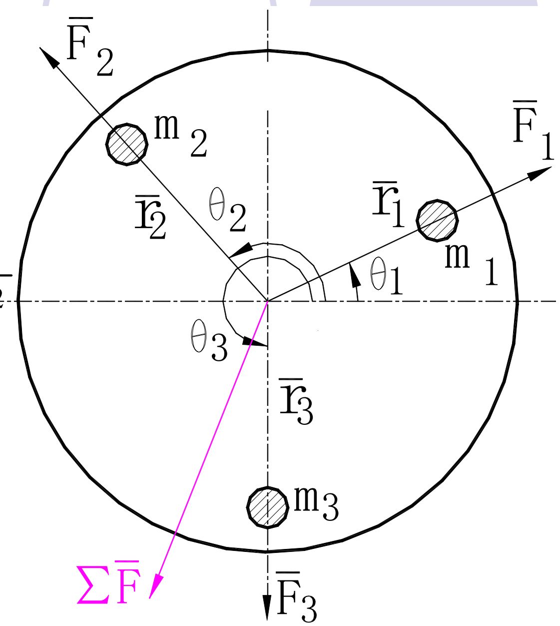
Static Balance:

The condition for the balancing of disk-like rotor : The vector sum of all inertia forces or the vector sum of all mass-radius products must be zero.

静平衡：各质量块的质径积的矢量和为零，或 $\sum \vec{F_i} = 0$

10.2.2 Calculation for the Balancing of a Disk-like Rotor

$$\begin{cases} m_C r_C \cos\theta_C + \sum m_i r_i \cos\theta_i = 0 \\ m_C r_C \sin\theta_C + \sum m_i r_i \sin\theta_i = 0 \end{cases}$$



$$m_C \cdot r_C = \sqrt{\left(\sum m_i r_i \cos\theta_i\right)^2 + \left(\sum m_i r_i \sin\theta_i\right)^2}$$

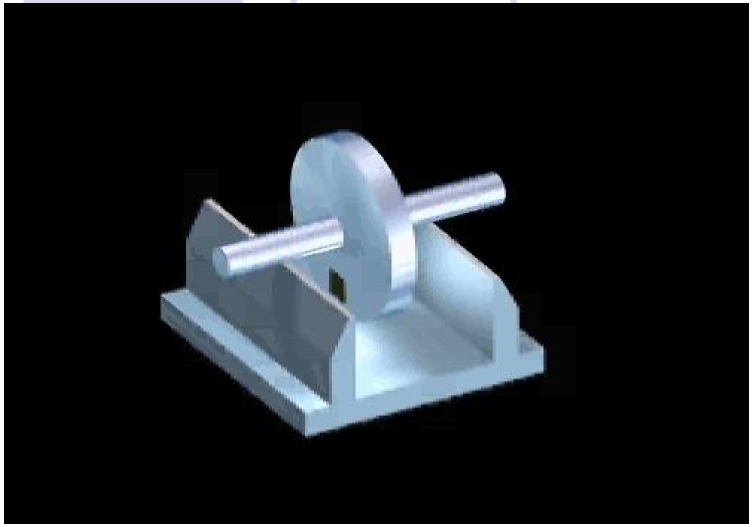
$$\theta_C = tg^{-1}\left(\frac{-\sum m_i r_i \sin\theta_i}{-\sum m_i r_i \cos\theta_i}\right)$$

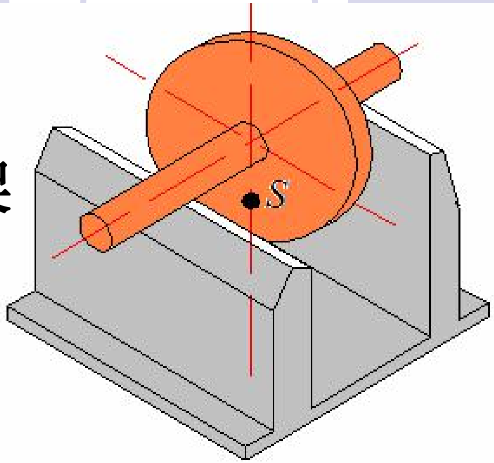
where θ_i represents the location angle of the mass *i*.

Note that the proper **qua**drant(象限) for the angle θ_C must be determined by the signs of both the **nu**merator(分子) and de**no**minator(分母) of the arctan function in Eq.

From the above, we can conclude that any number of masses in a disk-like rotor can be balanced by adding a single mass or removing a mass at an appropriate position.

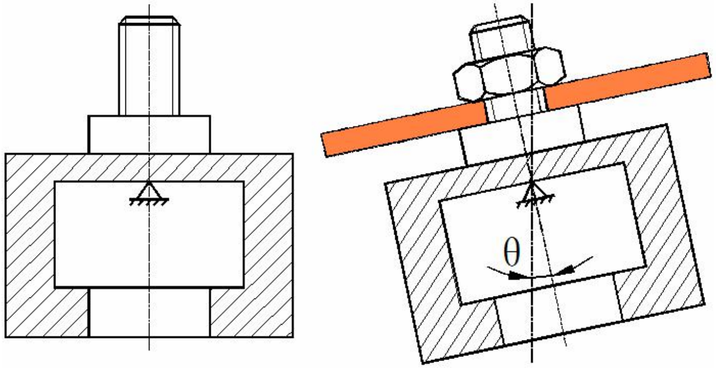
10.2.3 Static Balancing Machines





静平衡架

The correction by the static balancing machine must be made by trial and error, it will take much time to balance a rotor.



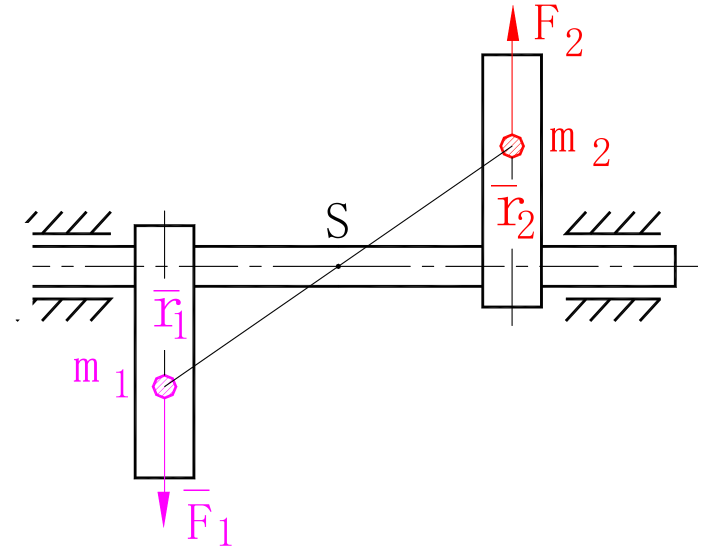
The static balancing machine shown in Fig. can indicate both the **mag**nitude and the location of imbalance at the same time.

10.3 Balancing of non-disk Rigid Rotors

10.3.1 Conditions for the Balancing of a Non-disk Rigid Rotor

$\frac{d}{b} \leq 5$ 质量分布不在同一回转面内

各部分质量的惯性力组成——空间力系



The conditions for the balancing of a non-disk rigid rotor are: ***Both the vector sum of all inertia forces and the vector sum of all moments of inertia forces about any point must be zero.***

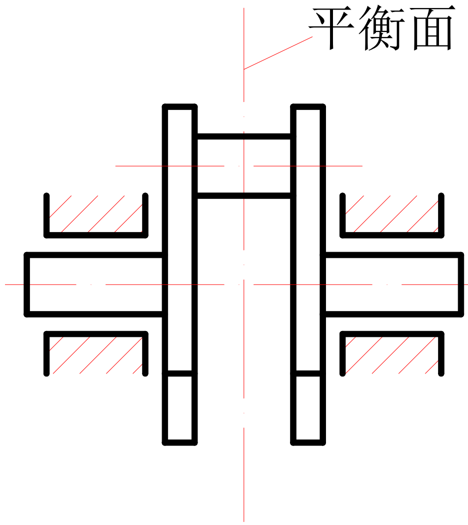
动平衡：转子离心力系的合力和合力矩都等于零

$$\sum \overline{F}_i = 0$$

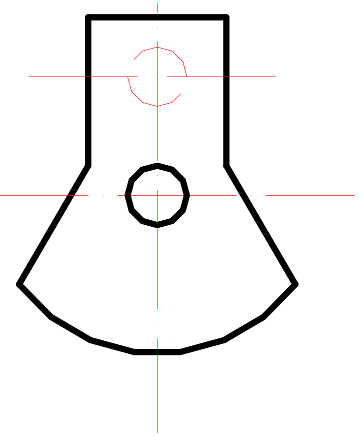
$$\sum \overline{M}_i = 0$$

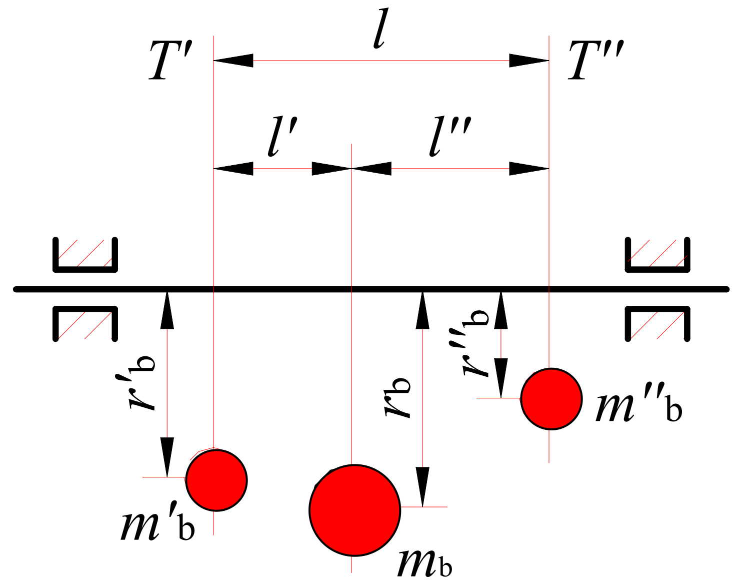
10.3.2 Resolution(分解) of Forces

The resultant force of F'_b and F''_b must be equal to and the resultant of the moments of F'_b and F''_b about any point must be equal to the moment of F about the same point.



例：曲轴的平衡



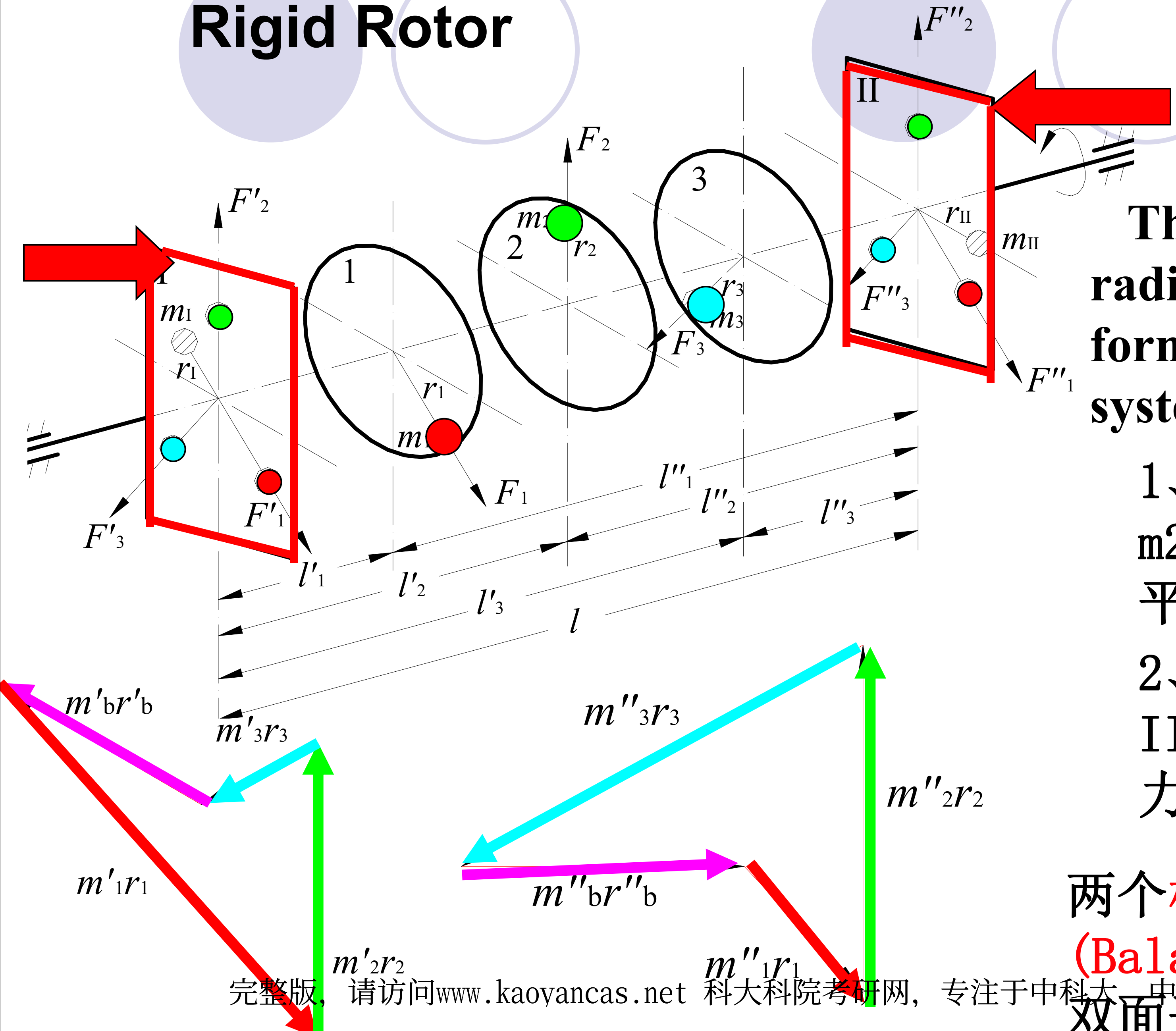


等效条件

$$\begin{cases} F'_b + F''_b = F_b \\ F'_b l' = F''_b l'' \end{cases} \Rightarrow \begin{cases} m'_b r'_b + m''_b r''_b = m_b r_b \\ m'_b r'_b \cdot l' = m''_b r''_b \cdot l'' \end{cases} \Rightarrow \begin{cases} m'_b r'_b = \dfrac{l''}{l} m_b r_b \\ m''_b r''_b = \dfrac{l'}{l} m_b r_b \end{cases}$$

若 $r'_b = r''_b = r_b$ $\quad m'_b = \dfrac{l''}{l} m_b$， $m''_b = \dfrac{l'}{l} m_b$

10.3.3 Calculation for the balancing of a Non-disk Rigid Rotor



The three mass-radius products form a spatial force system.

1、将质量m1，m2，m3代换到I平面和II平面

2、在I平面和II平面各自做力的平衡

两个校正平衡面(Balance planes)，双面平衡

比较：

静平衡 $\sum \overrightarrow{F_i} = 0$

动平衡：$\sum \overrightarrow{F_i} = 0$

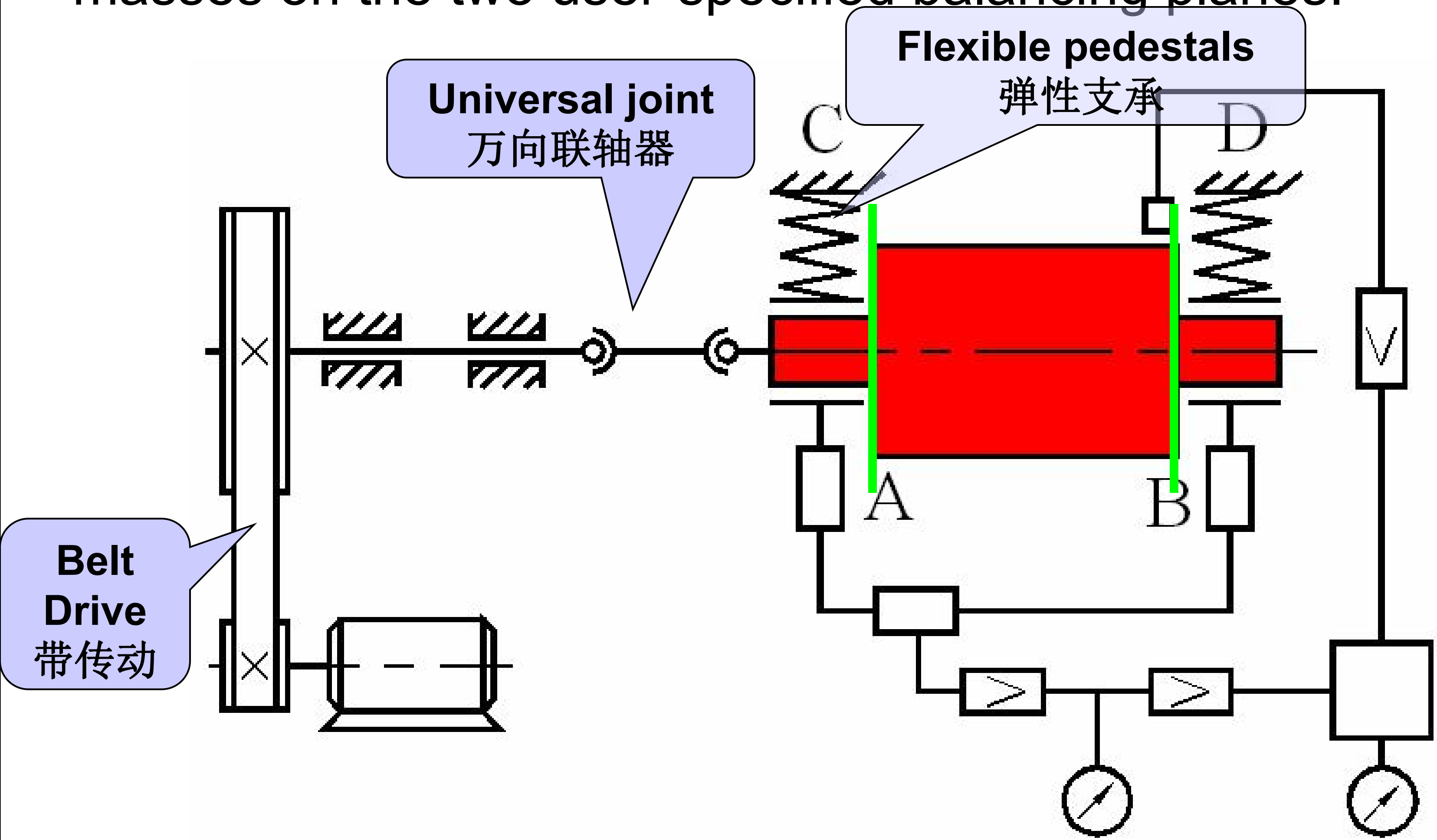
$\sum M_i = 0$

Static balancing is sufficient for disk-like rotors, while non-disk rigid rotors must also be dynamically balanced.

A dynamically balanced rotor is also statically balanced, but, in general, the converse is not true.经过动平衡的回转件一定是静平衡的，反之，静平衡的回转件不一定是动平衡的。

10.3.4 Dynamic Balancing Machines

The task of a dynamic balancing machine is to locate the magnitude and the angular position of these two e**qui**valent masses on the two user-specified balancing planes.



Dynamic balancing machine

10.4 Unbalancing Allowance(许用不平衡量) of Rotor

We cannot expect absolutely perfect balancing, because the resolution(分辨率) of any measurement instrument is limited. Some imbalance will remain in the rotor after balancing.

The amount of residual(剩余的) eccentricity that can be tolerated(允许的) is called *tolerable imbalance* (许用不平衡量).

Table 10-1

Recommended values of the allowable quality grade $G=[e\omega]$

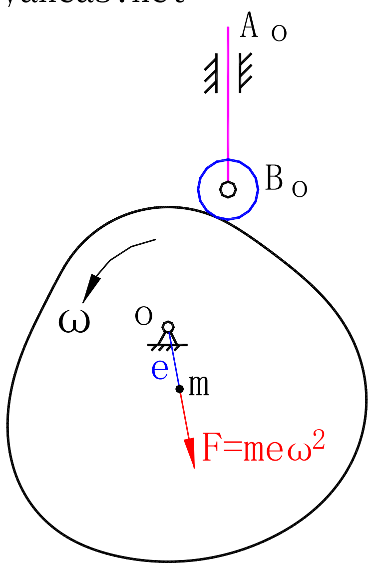
G(mm/s)	Rotor types
1600	Crankshaft assembly of large two-cycle engine.
6.3	Fans, flywheels, machine tools
1	Tape recorder(磁带录音机) and photograph drives, grinding machine(磨床) driver

回转件精度：$\frac{\text{不平衡质径积}}{\text{回转件质量}}\times\text{回转角速度}=e\cdot\omega=G$

Chapter 10

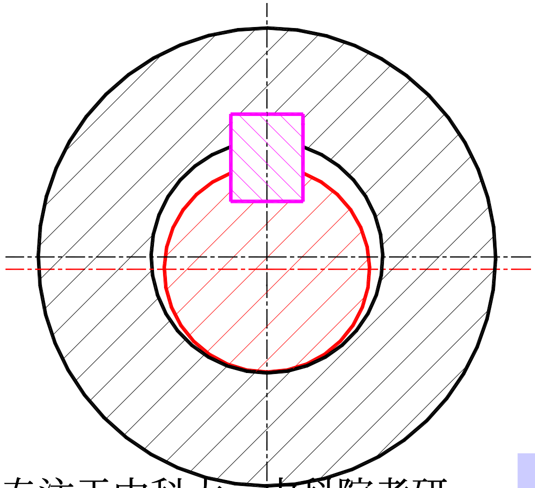
The End

The centre of mass of some machine elements, e.g. cams, may not coincide(重合) with their rotating centres because of the asymmetry(不对称) of the structure.



Even for symmetrical(对称的) machine elements, the centre of mass may still be eccentric(偏心的) because of uneven distribution of materials, errors in machining and also in casting(铸造) and forging(锻造).

Other errors may be caused by improper boring(镗孔), by keys and by assembly(装配).



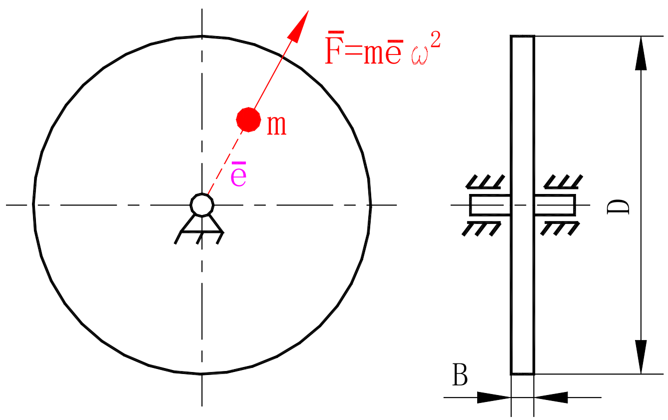
Rigid rotors (刚性转子)

The rotating frequency of the rotor is less than (0.6~0.7)n_{C1} (where: n_{C1} is the first resonant(共振) frequency of the rotor.

No deformation(变形)

(1) Disk-like rigid rotor

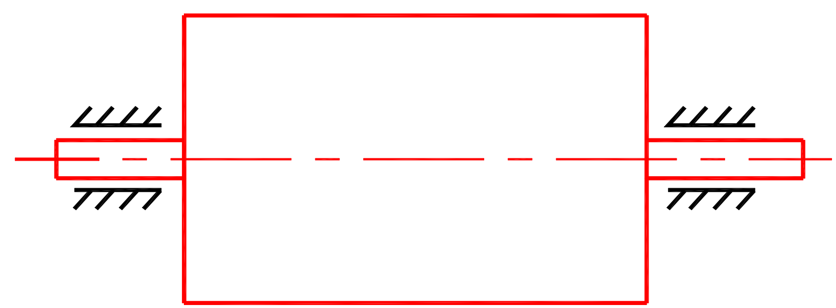
$$\frac{\text{axial dimensions B}}{\text{diameter D}} < 0.2$$



The masses of such rotors are assumed practically to lie in a common transverse plane.

(2) Non-disk rigid rotor

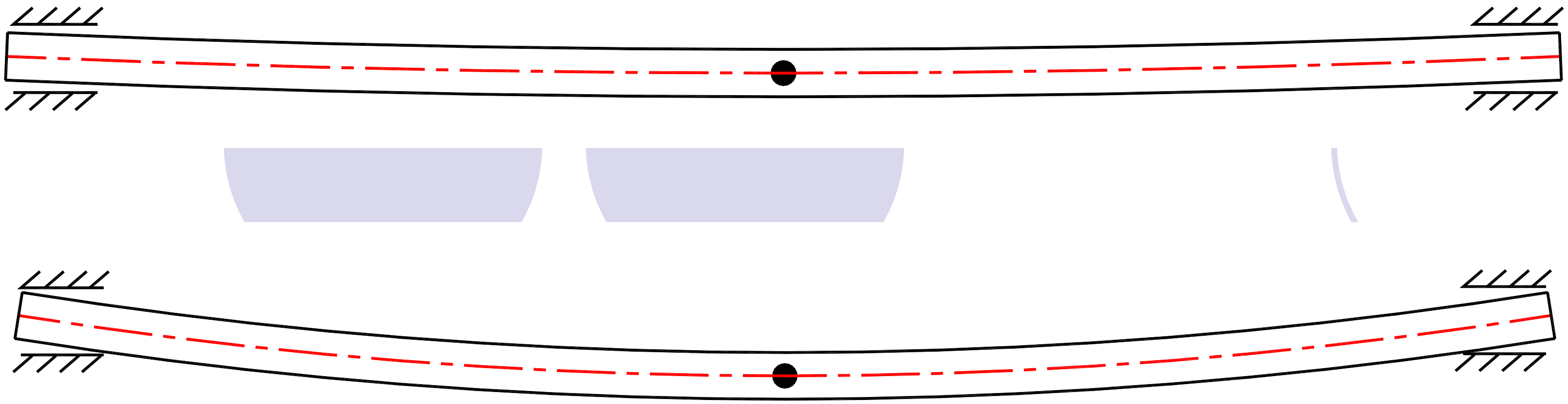
$$\frac{\text{axial dimensions B}}{\text{diameter D}} > 0.2$$



The masses of such rotors are assumed practically to lie in many transverse plane.

Flexible rotors (柔性转子)

The elastic deformation (变形) makes the eccentricity(偏心距) larger than the original one so that a new imbalance factor is added.

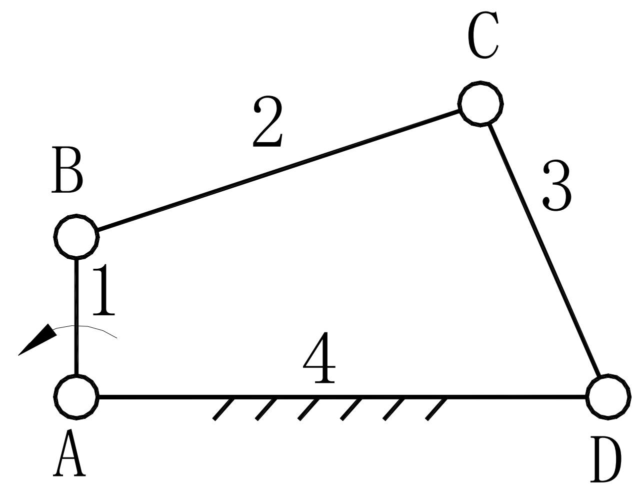


Balancing of mechanisms

The resultant(合成的) inertia force of all moving parts is equal to the net unbalanced force acting on the frame of a machine, which is referred to as the shaking force(振动力).

A resultant unbalanced moment acting on the frame, caused by the inertia forces and inertia moments of all moving parts, is called the shaking moment(振动力矩).

Although there exist many methods to make a linkage mechanism fully shaking force and/or fully shaking moment balanced, the results are usually unreasonable(不合理的).



Optimization(优化) methods can be used to reach a reasonable result.